SENSITIVITY ANALYSIS OF THE SHORT-TERM FORECASTS OF AN INTEGRATED HYDROMETEOROLOGICAL FORECAST SYSTEM

By
K. P. Georgakakos, C. Viswesvaran and W. F. Krajewski

Sponsored by:
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Department of the Army
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(Contract No. DACA89-87-K-0004)

IIHR Report No. 329
Department of Civil and Environmental Engineering
and
Iowa Institute of Hydraulic Research
The University of Iowa
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Executive Summary

An Integrated Hydrometeorological Forecast System (IHFS) that links quantitative precipitation prediction components with hydrologic runoff components and with real-time updating procedures was the subject of an extensive sensitivity analysis. The sensitivity of the IHFS stage forecasts with respect to: meteorological input (surface air temperature, pressure and dew-point temperature) time series, mean areal precipitation input time series, and observed stage time series, was examined. Data from three hydrologic basins of different hydromorphological characteristics were used in the sensitivity analysis. The results of the sensitivity analysis show that for flashy basins the meteorological input is very important for accurate predictions of flooding occurrence and magnitude. As the drainage basin response time becomes considerably greater than the forecast lead time, the sensitivity analysis showed that updating from rainfall and stage data is the dominant factor for accurate stage predictions. Updating from both rainfall and stage was shown to be necessary for robust IHFS predictions in situations with erroneous input data.

IHFS was installed on the Rock Island U.S. Army Corps of Engineers District on a HARRIS 800 computer. Software was written to interface IHFS with the hydrometeorological database maintained on the HARRIS. The sensitivity analysis of IHFS for the West Otter Creek at Kanawha, Iowa, basin detailed in this report was performed on the HARRIS computer. The IIHR Report No. 322 documents a User's Manual for IHFS. The IIHR Technical Note No. 6 presents the modifications made in IHFS that were necessary for its implementation onto the HARRIS 800 computer, and examples of interactive sessions.
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Jim Cramer of IIHR proved invaluable in the implementation of IHFS on the HARRIS computer and in the development of the software interface for accessing the HARRIS real-time hydrometeorological database (IIHR Technical Note No. 6). Shuguang Li, a graduate student in the Civil and Environmental Engineering Department of The University of Iowa, assisted with the preparation of the plots presented in the three Appendices of this report.
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SENSITIVITY ANALYSIS OF AN INTEGRATED
HYDROMETEOROLOGICAL FORECAST SYSTEM

I. INTRODUCTION AND REVIEW

A. Study Framework

The first hydrometeorological forecast systems suitable for use in flood and flash-flood prediction have appeared in the literature (Georgakakos and Hudlow, 1984, and Georgakakos, 1986a-b, 1987). They consist of precipitation, soil-moisture accounting, channel-flow routing and real-time updating components. They provide the framework within which hydrometeorological data of surface air temperature, humidity, pressure, precipitation and stage (or discharge), are used by physically-based models on the catchment scale to produce real-time flood and flash-flood predictions. These predictions can further be used as a basis for issuing flood warnings (Georgakakos, 1986c) or as a real-time input to operation procedures for reservoir systems (Georgakakos A. and Marks 1987).

Previous studies have examined the sensitivity of the predictions of hydrometeorological forecast systems to the precipitation component (Georgakakos 1986a-b), to the location of the station supplying the meteorological data (Georgakakos, 1987), and to the soil-moisture accounting and updating model components (Georgakakos, 1987). Those results are briefly summarized in the next section. The main theme of this report is the examination of the sensitivity of IHFS predictions to the input data. For this purpose a framework of sensitivity analysis is developed suitable for use with dynamical models and time-dependent input.

The next Chapter presents the basis of the IHFS formulation and design. Chapter III develops a methodology for sensitivity analysis and shows results of application of the methodology to three different hydrologic drainage basins in the United States. Conclusions are drawn in Chapter IV together with recommendations for future research and development. The results of this study can be used to determine the importance of the various groups of hydrometeorological data to real-time flood prediction. They can also provide guidelines for the design of effective real-time data quality control schemes.
and of observation-sensor networks to be used for forecasting and control of reservoir hydrosystems.

B. Results of Past Sensitivity Analyses

Georgakakos (1986a,b) established the improved performance of flood forecasting systems when precipitation prediction components are coupled to hydrologic runoff components via mass conservation equations and state-updating feedback. His results indicate improved performance in predicting the timing and magnitude of flood-peaks compared to the performance of traditionally used hydrologic prediction models (models without precipitation prediction components).

Sensitivity with respect to the location of the meteorological station supplying input data to the precipitation component was examined by Georgakakos (1987) for a recent event that caused excessive flooding in West Virginia. He found that for the case of updating from both stage and precipitation, the integrated hydrometeorological forecast system predictions were not significantly sensitive to the location of the meteorological station. Noticeable differences were observed in the predictions using input from various meteorological stations for cases of no updating and updating from only stage. The location of the station with respect to the storm moisture inflow was a significant factor in those cases.

A significant finding for cases of short-term (compared to catchment response time) flash flood prediction was that the predictions were insensitive to the soil-moisture accounting component of the system for all cases of updating examined (see Georgakakos, 1987). In addition, it was found that best results were obtained when real-time updating from both precipitation and stage data was performed. Perhaps more importantly, Georgakakos (1987) showed that in such a case, the predictions "...appear to be insensitive to the spatial configuration of the meteorological stations and to the runoff-generating model parameters." This last finding is particularly significant for the real time implementation of IHFS and for the design of sensor networks for localized flash flood predictions. It indicates that for robust behavior, real-time data from both precipitation and stage (or discharge) sensors are necessary.
II. INTEGRATED HYDROMETEOROLOGICAL FORECAST SYSTEM

A. Introduction

Stochastic-dynamic hydrological models have been used in the past for the real time forecasting of flood flows (e.g. Kitanidis and Bras, 1980a-b). In these models, precipitation was treated as an input with given mean and variance. With the advent of a stochastic-dynamic precipitation model (Georgakakos and Bras, 1984a-b) and of a procedure for the spatial interpolation of the meteorological input, precipitation could be treated as a dynamic process with a state variable that could be added on to the state variables of the hydrologic models. A stochastic-dynamic hydrometeorological model was the result (Georgakakos and Hudlow, 1985, Georgakakos, 1986a-b, Georgakakos, 1987).

Some of the characteristics of the stochastic dynamic hydrometeorological model that make it particularly useful in real-time flood and flash flood forecasting are:

1. Extended forecast lead time, especially in cases of flash floods, because of the quantitative precipitation forecast capability.

2. Better distribution of the precipitation volume in the various soil zones because of the existence of a dynamic equation for the precipitation state.

3. Improved modeling of uncertainty through the incorporation of a precipitation state in the state vector.

4. Capability for updating the upper soil states from only precipitation observations.

Real time operation of the stochastic-dynamic hydrometeorological model for the prediction of flood flows involves the following steps:

1. Collection of meteorological data from stations located nearby the watershed of interest.

2. Spatial interpolation of the meteorological data and determination of averaged values for the watershed.

3. Using a set of initial mean and covariance values for the states of the hydrometeorological model and the interpolated input values, the model dynamic equations and the state estimator variance equations are solved for the forecast lead time interval. Predictions of the watershed-average preci-
pitation rate and of the flow at the watershed outlet are obtained from the predicted states of the system.

4. Collection of raingage data and stage data at observation time. Computation of watershed mean areal precipitation and discharge from collected data.

5. Use of the state estimator update equations for the determination of improved estimates for the mean and covariance of the states of the hydrometeorological model at the observation time.

6. Use of the updated state mean and covariance as initial conditions for the next time step that repeats steps 1 through 5.

Georgakakos (1986a-b) used the modified Sacramento hydrologic model and the Georgakakos and Bras (1982) flood routing model as the soil and channel components of the hydrometeorological model. Georgakakos (1987) successfully used a simple Antecedent Precipitation Index (API) model as the runoff generating model together with the previously mentioned flood routing model for flash flood prediction. In the following we present the formulation of the latter hydrometeorological model that forms the basis of the Integrated Hydrometeorological Forecast System (IHFS).

B. Precipitation Component

The Georgakakos and Bras (1984a-b) station precipitation model was used to generate precipitation predictions based on surface air pressure, temperature and dew-point temperature data. The model equations can be written in the following form:

$$\frac{dx_p(t)}{dt} = f[u(t)] + h[u(t)] x_p(t)$$  \hspace{1cm} (II.1)

and

$$y_p(t_k) = \Delta t \phi[u(t_k)] x_p(t_k); \quad k = 1, 2, \ldots$$  \hspace{1cm} (II.2)

where $x_p(t)$ is the condensed water equivalent mass (or volume) of water at time $t$ in a unit area column that extends from the bottom to the top of the clouds (in kilograms per square meter or millimeters per square meter), $u(t)$ is the vector of the precipitation model inputs (i.e., surface air temperature, pressure, and dew-point temperature) at time $t$, $\{h[u(t)] x_p(t)\}$ is the
outflow mass (or volume) rate of condensed water equivalent from the cloud column at time \( t \) (in kilograms per square meter per second or in millimeters per square meter per second), \( f[u(t)] \) is the inflow mass (or volume) rate of condensed water equivalent into the cloud column at time \( t \) (in kilograms per square meter per second or in millimeters per square meter per second). Outflow is due to precipitation or local cloud-top anvil formation, while inflow is due to condensation and air mass ascent. Subcloud evaporation reduces the precipitation mass that reaches the ground to a value smaller than the one obtained at the cloud bottom level. The precipitation mass (or volume) reaching the ground is collected between time instants \( t_k \) and \( t_{k+1} \), for \( k = 1, 2, \ldots \), \( (\Delta t = t_{k+1} - t_k) \) and is represented by \( y_p(t_k) \) (in kilograms per square meter /\( \Delta t \) or millimeters per square meter /\( \Delta t \)). The instantaneous precipitation rate at ground level is given by \([\phi[u(t_k)] x_p(t_k)] \) (in kilograms per square meter per second or millimeters per square meter per second).

The details of model derivation and expressions for \( f(\cdot) \), \( h(\cdot) \), and \( \phi(\cdot) \) are given by Georgakakos and Bras (1984a-b). The model is based on a convective parameterization of the vertically averaged updraft velocity as a function of the surface-air meteorological model input variables:

\[
v = \epsilon_1 \sqrt{c_p (T_m - T'_s)}
\]  

(II.3)

where \( \epsilon_1 \) is a model parameter, proportional to the square root of the ratio of kinetic to maximum thermal energy per unit mass of ascending air; \( c_p \) is the specific heat of dry air under constant pressure; \( T_m \) is the cloud temperature at a certain pressure level \( p' \); and \( T'_s \) is the corresponding potential ambient air temperature, determined from surface air meteorological data using adiabatic ascent of air from the ground surface. The level \( p' \) is defined by

\[
p' = p_s - \frac{1}{4} (p_s - p_t)
\]  

(II.4)

where \( p_s \) is the cloud bottom pressure computed from heat-adiabatic ascent and \( p_t \) is the cloud top pressure.

Using the cloud updraft velocity \( v \), the condensation inflow rate \( f[u(t)] \) was obtained by
where $\Delta W$ is the mass of liquid water resulting from condensation during the pseudo-adiabatic ascent of a unit mass of moist air, $\rho_m$ is a vertically averaged (from cloud top to cloud bottom) density of moist air, and $dA$ represents the unit area measure.

Cloud microphysics transforms the discrete mass flux of the falling precipitation drops to the continuous precipitation rate through the use of an exponential particle size distribution:

$$n(D) = N_0 \exp \left( - \frac{D}{\epsilon_2} \right)$$

where $N_0$ is a parameter expressed as a function of the model state variable $x_p$, $D$ is the drop diameter and $\epsilon_2$ is a free model parameter representing the average diameter of hydrometeors in the cloud column.

Since the emphasis on flood and, especially, flash flood prediction is in areas of the order of $10^2 - 10^3$ km$^2$, using the station precipitation model requires surface air meteorological data sampled at points separated by distances of similar spatial scale. On the average the surface meteorological data stations in the United States have separation distances greater than 100 km. An interpolation procedure is then necessary to provide input to the precipitation model for all the possible locations of interest. Georgakakos [1986a] developed and tested such an interpolation procedure. The variables interpolated were surface air temperature, pressure, and dew-point temperature. The interpolation accounts for altitude-varying terrain. For flat terrain the procedure reduces to a linear spatial interpolation (weights inversely proportional to distance). Average interpolation errors of about 1$^\circ$K were obtained from tests in Oklahoma and errors of about 2$^\circ$K were obtained from tests in Montana for the surface air temperature variables. The pressure-variable errors were about 1 mbar. Station distances were greater than 150 km in the test cases. Complemented by the input interpolation procedure, the precipitation model can be used to predict mean areal precipitation for any drainage basin given only surface air meteorological data at a few stations.
Several flash floods are the result of orographic enhancement of precipitation followed by the resultant fast-moving flood waves on steep slopes. The previously described spatial interpolation methodology can produce estimates of surface air meteorological variables in mountainous terrain. Through the terrain variation of the aforementioned meteorological variables that are used as an input to the precipitation prediction model, the orographic effects are partially taken into account. The most important effect of orography, however, that is, the enhancement of the updraft velocity and of the mass influx into the clouds, should be simulated by the model if accurate prediction of precipitation in mountainous areas are to be obtained.

Several studies have examined the issues concerning the modeling of orographic enhancement, given operationally available meteorological data (e.g., Elliott and Shaffer, 1962; Bell, 1978; Rhea, 1978). The basic idea is that the updraft velocity enhancement due to orography is equal to the inner product of the two vectors: the horizontal wind vector $\hat{W}$ and the local topographic gradient $\vec{V_H}$.

$$V_o = \hat{W} \cdot \vec{V_H} \quad (II.7)$$

where $V_o$ is the magnitude of the orographic component of the updraft velocity. The operational models presented in the literature to date vary in the ways they treat the vertical distribution of the updraft velocity enhancement and the conditions under which enhancement can occur.

In an ongoing research effort the stochastic-dynamic model of the previous sections has been modified to incorporate orographic enhancement of the updraft velocity. When only surface meteorological data are available (e.g., in cases when the forecast lead time is less than 6-hours), (II.7) is used to estimate the surface updraft enhancement and a linear reduction of the orographic updraft velocity component with height is assumed with the vanishing level at the cloud top level. No orographic enhancement is computed in cases when the horizontal surface wind is less than 2.5 m s$^{-1}$ or when the surface relative humidity is less than 65%.

Thus given that the aforementioned criteria for the existence of orographic enhancement are fullfilled, the vertically averaged updraft velocity (see equation (II.3)) is incremented by the amount $\frac{1}{2} V_o$, while the condensa-
tion input rate at cloud bottom (see equation (II.5)) is incremented by the amount \( \Delta W_0 \cdot V_0 \cdot dA \).

The problem of spatial interpolation of the wind vector arises in cases where orographic enhancement is significant. In this work the wind vector observed at an upwind station is assumed to be representative of the wind at the drainage basin of interest.

In cases where the drainage basin is divided into more than one orographic zone, a precipitation prediction model of the type described previously, was formulated for each orographic zone. Then, the drainage basin mean areal precipitation at time \( t_k \) \( \left[ \bar{y}_p(t_k) \right] \) was computed by

\[
\bar{y}_p(t_k) = \sum_{i=1}^{L} \frac{A_i}{A} \cdot y_{pi}(t_k)
\]

where \( A_i \) is the area of the \( i \)th orographic zone, \( A \) is the total area of the drainage basin, and \( y_{pi}(t_k) \) is the precipitation prediction at time \( t_k \) computed for the \( i \)th orographic zone. The areas of the \( L \) orographic zones satisfy

\[
\sum_{i=1}^{L} A_i = A
\]

\[ \text{(II.9)} \]

**C. Runoff-Generating Component**

Previous work in the area of integrated hydrometeorological modeling (Georgakakos, 1986a, 1987) has used conceptual, multicompartment hydrologic models for the computation of the channel inflow. Thus, the Sacramento soil-moisture accounting model (Burnash et al., 1973; Peck, 1976) was written in a time-continuous state-space form in order to conform with the other components of the hydrometeorological model. Given the short data records of most basins of small areal extent and the fact that it is widely used by most of the U.S. River Forecast Centers (RFCs), a simple, computationally efficient antecedent precipitation index (API) model was used in this work following Georgakakos (1987).

The analytical form of the model can be written as

\[
\text{API}(t_k) = \text{API}(t_{k-1}) \cdot q^{\Delta t/24} + \bar{y}_p(t_k)
\]

\[ \text{(II.10)} \]
\[ u(t_k) = c + [a + d \cdot s(t_k)] \cdot \exp [-b \cdot API(t_k)] \]  \hspace{1cm} (II.11)

\[ u_c(t_k) = \left[ \left( \sum_{i=1}^{i=k} y_p(t_i) \right)^n + u^n(t_k) \right]^{1/n} - u(t_k) \]  \hspace{1cm} (II.12)

where \( API(t_k) \) is the antecedent precipitation index at time \( t_k \), \( s(t_k) \) is a predetermined seasonal index at time \( t_k \), \( t_1 \) is the time of storm initiation, \( u_c(t_k) \) is the generated surface runoff (channel inflow) at time \( t_k \), \( q \) is an API daily reduction factor taken equal to 0.9, and \( c, a, d, b, n \) are model parameters. The precipitation variable \( y_p(t_k) \) was defined in (II.8).

Various forms of the API model are currently in use at the various U.S. RFCs. Model differences arise as a result of adaptation to local conditions. The present mathematical development utilizes the basic model, described by (II.10) through (II.12). Betson et al., (1969) give a set of parameters fitted to basins of various areas and hydrologic characteristics.

**D. Channel Routing Component**

The conceptual, nonlinear, reservoir-type flood-routing model of Georgakakos and Bras (1982) was used to propagate the flood wave downstream, up to the point or points of interest. The model equations are

\[ \frac{dS_i(t)}{dt} = u_c(t_k) - \beta S^m_i(t); \hspace{0.5cm} t_k < t < t_{k+1}; \hspace{0.5cm} k = 1, 2, \ldots \]  \hspace{1cm} (II.13)

\[ \frac{dS_i(t)}{dt} = \beta S^m_{i-1}(t) - \beta S^m_i(t); \hspace{0.5cm} t_k < t < t_{k+1}; \hspace{0.5cm} k = 1, 2, \ldots; \hspace{0.5cm} i = 2, 3, \ldots, n_c \]  \hspace{1cm} (II.14)

where \( S_i(t) \) is the water volume stored in the \( i \)th channel reach (conceptual reservoir) at time \( t \) (in millimeters per square meter); \( \beta \), \( m \), and \( n_c \) are model parameters with \( n_c \) representing the number of channel reaches.

The original model formulation calls for a different value of the coefficient \( \beta \) for each of the \( n_c \) conceptual reservoirs. Given the lack of data for calibration in most flash flood prone areas, a common value of the coefficient \( \beta \) was used in this work.
The discharge outflow \( y_c(t_k) \) from the \( n_c \)th reservoir at time \( t_k \) is the routing model output, and it is given by

\[
y_c(t_k) = \beta S_{n_c}^m(t_k); \quad k = 1, 2, \ldots
\]  

(II.15)

A stage-discharge table (or function) for a particular stream location can be used to convert the discharge \( y_c(t_k) \) to water depth (stage) \( h_c(t_k) \), if water depth is of interest.

E. State Updating Component

Past studies (e.g. Kitanidis and Bras, 1980a-b; Georgakakos and Bras, 1982; Georgakakos and Bras 1984a-b; Georgakakos, 1986a-b) have well established that the use of modern estimation theory techniques to process system output observations: (1) improves the performance of conceptual, hydrologic, and hydrometeorologic deterministic models, and (2) provides a framework for the statistically consistent determination of the prediction errors of these models, given statistical description of the errors in the model input, parameters, and structure. The extended Kalman filter (EKF) (see Gelb (1974) or Bras and Rodriguez-Iturbe (1985) for descriptions) has proven useful as a state estimator in cases when the model under study involves nonlinear functions of the state variables. In the following, a short presentation of the EKF equations is given for the models of the integrated hydrometeorological forecast system.

Define the state vector \( \mathbf{x}(t) \) by

\[
\mathbf{x}(t) = \begin{bmatrix}
x_1(t) \\
p_1 \\
\vdots \\
x_L(t) \\
p_L \\
S_1(t) \\
\vdots \\
S_{n_c}(t)
\end{bmatrix}
\]  

(II.16)

10
Define also the vector nonlinear function \( \mathbf{F}(\mathbf{x}, t) \) by

\[
\mathbf{F}[\mathbf{x}(t), t] = \begin{bmatrix}
    f_1[u_1(t)] + h_1[x_1(t)] \cdot x_{p1} \\
    \vdots \\
    f_L[u_L(t)] + h_L[x_L(t)] \cdot x_{pL} \\
    u_c(t_k) - \beta s_{n}^m(t) \\
    \vdots \\
    \beta s_{n}^{m-1}(t) - \beta s_{n}^m(t)
\end{bmatrix}; \quad t_k < t < t_{k+1}, \quad k = 1, 2, \ldots \tag{II.17}
\]

In the latter two equations, subscripts 1 to L refer to orographic zones.

Then, the dynamic equation of the flash flood prediction system becomes

\[
\frac{d\mathbf{x}(t)}{dt} = \mathbf{F}[\mathbf{x}(t), t] \tag{II.18}
\]

To account for model structure, model parameters and model input errors, a random error term represented by the vector \( \mathbf{w}(t) \) is added to the right-hand side of (II.18), resulting in

\[
\frac{d\mathbf{x}(t)}{dt} = \mathbf{F}[\mathbf{x}(t), t] + \mathbf{w}(t) \tag{II.19}
\]

An observation vector \( \mathbf{Z}(t_k) \) is defined with elements \( Z_{p1}(t_k), \ldots, Z_{pL}(t_k) \) and \( Z_c(t_k) \) representing the precipitation observations at each orographic zone (if any exist) at time \( t_k \) and the stream outlet discharge (or stage) observation at the same time. Denoting by \( v_p(t_k) \) and \( v_c(t_k) \) two random sequences that represent the observation errors of precipitation and discharge (stage), respectively, a system observation equation can be written as follows:

\[
\mathbf{Z}(t_k) = \mathbf{G}[\mathbf{x}(t_k), t_k] + \mathbf{v}(t_k); \quad k = 1, 2, \ldots \tag{II.20}
\]

where \( \mathbf{G}[\mathbf{x}(t_k), t_k] \) is defined by
\[ G(\mathbf{x}(t_k), t_k) = \begin{bmatrix} \Delta t \phi_1(u_1(t_k)) \mathbf{x}(t_k) \\ \vdots \\ \Delta t \phi_L(u_L(t_k)) \mathbf{x}(t_k) \\ \beta S m_{nc}(t_k) \end{bmatrix} \]  

(II.21)

and \( \mathbf{v}(t_k) \) is defined by

\[ \mathbf{v}(t_k) = \begin{bmatrix} v_p(t_k) \\ \vdots \\ v_p(t_k) \\ v_c(t_k) \end{bmatrix} \]  

(II.22)

with \( L \) elements equal to \( v_p(t_k) \).

Equations (II.19) and (II.20) represent the state-space form of the models that constitute the hydrometeorological forecast system. Because of nonlinearities present in the channel-routing model, the state-space equation is nonlinear in the state \( \mathbf{x}(t) \). In case stage is observed, the "observed" discharge is computed by the stage-discharge function corresponding to the location of interest.

The random model-error vector process \( \mathbf{w}(t) \) is assumed to have a mean equal to zero and a covariance parameter (spectral density) matrix denoted by \( Q \). The random observation-error vector sequence \( \mathbf{v}(t_k) \) is assumed to have a mean equal to zero and a covariance matrix denoted by \( \mathbf{R}(t_k) \).

In this work, \( Q \) and \( \mathbf{R}(t_k) \) are diagonal matrices. The diagonal elements of matrix \( \mathbf{R}(t_k) \) are

\[ r_{p_i}(t_k) = (1.0 \text{ mm}^2 \text{ hr}^{-2}); \quad i = 1, \ldots, L \]

and

\[ r_c(t_k) = \{[0.1 Z_c(t_k) + 0.1]^2 \text{ mm}^2 \text{ hr}^{-2}\} \]

where

\[ \mathbf{R}(t_k) = \begin{bmatrix} r_p(t_k) & 0 \\ \vdots & r_p(t_k) \\ 0 & r_p(t_k) \\ r_c(t_k) & \end{bmatrix} \]  

(II.23)
The above values reflect the expected errors in measuring rainfall and discharge in real time.

The elements of the \( Q \) matrix are system-free parameters and need to be estimated from observed data.

Note that in the previous formulation it was assumed that precipitation observations exist for all \( L \) orographic zones. In practice, this will rarely be the case. However, the previous state-space formulation can be used in other cases by eliminating the rows of \( Z(t_k) \), \( C(\mathbf{x}(t_k), t_k) \), \( v(t_k) \), and rows and columns of \( \mathbf{R}(t_k) \), corresponding to those orographic zones without observed data.

Define the \( ij \)th element of the model-derivatives matrix \( \mathbf{N}_F[\mathbf{x}_o(t), t] \) by

\[
\{\mathbf{N}_F[\mathbf{x}_o(t), t]\}_{ij} = \frac{\delta F_i[\mathbf{x}_o(t), t]}{\delta x_j(t)} \quad (II.24)
\]

where \( F_i(\cdot) \) is the \( i \)th element of the vector function \( \mathbf{F}(\cdot) \). Similarly, define the \( ij \)th element of the observation-derivatives matrix \( \mathbf{N}_G[\mathbf{x}_o(t), t] \) by

\[
\{\mathbf{N}_G[\mathbf{x}_o(t), t]\}_{ij} = \frac{\delta G_i[\mathbf{x}_o(t), t]}{\delta x_j(t)} \quad (II.25)
\]

where \( G_i(\cdot) \) is the \( i \)th element of vector function \( \mathbf{G}(\cdot) \). In (II.24) and (II.25), \( x_o(t) \) is a nominal value of the state vector \( \mathbf{x}(t) \).

The state estimator equations for the propagation of the state mean vector \( \mathbf{\hat{x}}(t) \) and state covariance matrix \( \mathbf{P}(t) \) between observations are given by

\[
\frac{d\mathbf{\hat{x}}(t/t_k)}{dt} = \mathbf{F}[\mathbf{\hat{x}}(t/t_k), t] \quad ; \quad t_k < t < t_{k+1} \quad (II.26)
\]

and

\[
\frac{d\mathbf{P}(t/t_k)}{dt} = \mathbf{N}_F[\mathbf{\hat{x}}(t/t_k), t] \cdot \mathbf{P}(t/t_k) \\
+ \mathbf{P}(t/t_k) \mathbf{N}_F^T[\mathbf{\hat{x}}(t/t_k), t] \cdot \mathbf{Q} \cdot \mathbf{P}(t/t_k) \quad ; \quad t_k < t < t_{k+1} \quad (II.27)
\]
where the representation \( \hat{x}(t_k+1/t_k) \) denotes the best estimate, given observations of system output up to and including time \( t_k \), and superscript \( T \) denotes transpose of a vector or matrix quantity.

The equations for the updating of the state second moments, given a new observation at time \( t_{k+1} \), are

\[
\hat{x}(t_{k+1}/t_k) = \hat{x}(t_{k+1}/t_k) + K(t_{k+1}) \cdot (\Sigma(t_{k+1}) - G[\hat{x}(t_{k+1}/t_k), t_{k+1}])
\]

\[
\Sigma(t_{k+1}/t_k) = (I - K(t_{k+1}) \cdot N_G[\hat{x}(t_{k+1}/t_k), t_{k+1}]) \Sigma(t_{k+1}/t_k)
\]

with \( K(t_{k+1}) \) defined by

\[
K(t_{k+1}) = \Sigma(t_{k+1}/t_k) N_G \cdot T[\hat{x}(t_{k+1}/t_k), t_{k+1}] \cdot Q^{-1}_v(t_{k+1})
\]

\[
Q_v(t_{k+1}) = N_G[\hat{x}(t_{k+1}/t_k), t_{k+1}] \cdot \Sigma(t_{k+1}/t_k) N_G \cdot T[\hat{x}(t_{k+1}/t_k), t_{k+1}] + R(t_{k+1})
\]

where \( I \) is the diagonal matrix of dimension equal to the state vector dimension with diagonal elements equal to 1 and off-diagonal elements equal to zero, and \( Q^{-1}_v \) denotes the inverse of the square symmetric matrix \( Q_v \).

The system (II.26) through (II.31) can be used iteratively in real time to process the observations in \( Z(t_k) \) as they become available. The quantities \( \hat{x}(t_{k+1}/t_k) \) and \( \Sigma(t_{k+1}/t_k) \) are the state vector best (in a minimum variance sense) estimate at time \( t_{k+1} \) and the associated error covariance matrix, respectively. Given initial values \( \hat{x}(0) \) and \( \Sigma(0) \) of these quantities at initial time \( t_0 \), the state estimator combines system equations with uncertainty estimates for observations and models to arrive at minimum variance predictions.

It is noted that given \( \hat{x}(t_{k+1}/t_k) \) and \( \Sigma(t_{k+1}/t_k) \), the best prediction of the system observables at time \( t_{k+1} \), given information up to and including time \( t_k \), is

\[
\hat{z}(t_{k+1}/t_k) = G[\hat{x}(t_{k+1}/t_k), t_{k+1}]
\]

(II.32)
and the associated error covariance matrix is

\[
\Sigma(t_{k+1}/t_k) = N_\Sigma \{ \hat{X}(t_{k+1}/t_k), t_{k+1} \} \Sigma(t_{k+1}/t_k) \cdot N_\Sigma \{ \hat{X}(t_{k+1}/t_k), t_{k+1} \}^T
\]  

(II.33)

Denote by \( \Sigma_{q}(t_{k+1}/t_k) \) the last diagonal element of matrix \( \Sigma_{q}(t_{k+1}/t_k) \), corresponding to the variance of the discharge output variable, and by \( \hat{Z}(t_{k+1}/t_k) \) the last element of \( \hat{Z}(t_{k+1}/t_k) \), corresponding to the best estimate (prediction) of discharge. Assuming Gaussian distribution of the discharge prediction errors, with moments \( \Sigma_{q}(\cdot) \) and \( \Sigma_{q}(\cdot) \), and given a discharge (or stage) threshold \( q_c \) above which flooding would occur, one can compute the probability PFO that flooding would occur at time \( t_{k+1} \), given information up to and including time \( t_k \), as

\[
PFO(t_{k+1}) = \int_{q_c}^{+\infty} f_N[\hat{Z}_{q}(t_{k+1}/t_k), \Sigma_{q}(t_{k+1}/t_k)] \, dz 
\]  

(II.34)

where \( f_N[\hat{Z}_{q}(\cdot), \Sigma_{q}(\cdot)] \) is the probability density function of the Gaussian distribution.

**F. Real-Time Coupling**

It is desired to identify the means by which meteorological and hydrological components are coupled in stochastic-dynamic hydrometeorological models. Real-time coupling of the hydrological with the meteorological components is illustrated in the following by way of analysis of the state estimator equations for a headwater drainage basin hydrometeorological model.

Define by \( \dot{X}_p \) the \( n_1 \)-dimensional vector of the precipitation model states, and by \( \dot{X}_h \) the \( n_2 \)-dimensional vector of the hydrological model states. Dependence on time \( t \) is not explicitly shown for notational convenience; it is noted however that such dependence exists and in most cases it is implicit through dependence on input data.

Form the composite \( n \)-dimensional vector \( \dot{X} \) as:

\[
\dot{X} = \begin{bmatrix} \dot{X}_p \\ - \dot{X}_h \end{bmatrix} \quad \text{with} \quad n_1 + n_2 = n 
\]  

(II.35)
Assume that the precipitation model dynamic equations are:

\[
\frac{dx}{dt}^p = f_p(x_p; a_p, u_p) \tag{II.36}
\]

and the hydrological model dynamic equations are:

\[
\frac{dx}{dt}^h = f_h(x_h, x_p; a_h, a_p, u_p, u_h) \tag{II.37}
\]

with attendant observation equations,

\[
z^p = h^p(x_p; a_p, u_p) \tag{II.38}
\]

\[
z^h = h^h(x_h; a_h) \tag{II.39}
\]

where

- \(t\): time;
- \(f_p(\cdot)\): functional form of the precipitation model temporal state derivative vector;
- \(f_h(\cdot)\): functional form of the hydrological model temporal state derivative vector;
- \(h^p(\cdot)\): functional form of the algebraic equation that describes the relationship between precipitation state and precipitation observations;
- \(h^h(\cdot)\): functional form of the algebraic equation that describes the relationship between hydrological-model state and outflow observations;
- \(a_p\): \(k_1\)-dimensional vector of precipitation model parameters;
- \(a_h\): \(k_2\)-dimensional vector of hydrological model parameters;
\( \mathbf{u}_p \) : \( \ell_1 \)-dimensional vector of precipitation model input;

\( \mathbf{u}_h \) : \( \ell_2 \)-dimensional vector of hydrological model input;

\( \mathbf{z}_p \) : \( m_1 \)-dimensional vector of precipitation observations;

\( \mathbf{z}_h \) : \( m_2 \)-dimensional vector of drainage basin outflow observations.

Dependence on input and parameter vectors is explicitly shown in the previous formulation based on the specific models used in Georgakakos (1986a,b, 1987). The analysis to follow, however, is independent of the specific models used.

Making use of the composite vector \( \mathbf{x} \), the state-space form of Equations (II.36) through (II.39) can be written in a manner analogous to Eqs. (II.19) and (II.20), with

\[
\Phi \hat{f} = \begin{bmatrix} \mathbf{f} \\ \mathbf{P}_p \\ \mathbf{h} \end{bmatrix}
\]

(II.40)

\[
\mathbf{G} \hat{\mathbf{h}} = \begin{bmatrix} \mathbf{h} \\ \mathbf{P}_p \\ \mathbf{h} \end{bmatrix}
\]

(II.41)

\[
\mathbf{a} = \begin{bmatrix} \mathbf{a} \\ \mathbf{a}_p \\ \mathbf{u} \end{bmatrix}
\]

(II.42)

\[
\mathbf{u} = \begin{bmatrix} \mathbf{u} \\ \mathbf{P}_p \\ \mathbf{h} \end{bmatrix}
\]

(II.43)

\[
\mathbf{z} \hat{\mathbf{z}} = \begin{bmatrix} \mathbf{z} \\ \mathbf{P}_p \\ \mathbf{h} \end{bmatrix}
\]

(II.44)

The filter Equations (II.26) through (II.31) can be used for propagation and updating of the state vector between and at the times of observation. Coupling of the meteorological with the hydrologic components is realized firstly in Equations (II.36) and (II.37) and secondly in Equations (II.28) through (II.31). In the former set of equations coupling is due to the
conservation of mass law: the output mass flux of the precipitation model is equal to the mass flux input into the hydrological component. It is a one-way coupling: from the precipitation to the hydrologic model. The latter set of Equations couples the precipitation components with the hydrological components through the feedback terms in Equations (II.28) and (II.29). Namely, terms of the type:

\[ K(\hat{x} - h(\hat{x};\alpha,\mu)) \]

and

\[ K_{hp} \]

where in the notation of Eqs. (II.28) through (II.31), \( Z \Delta z, G \Delta H, N_G \Delta H, \Sigma(t_{k+1}|t_k) \Delta p \).

Next, we partition the composite vectors and matrices into their components corresponding to the precipitation and hydrological processes in order to gain insight into the factors of the feedback coupling in the state mean.

At first, partition matrix \( K \) as follows:

\[ K = \begin{bmatrix} K_{pp} & \cdots & K_{ph} \\ \vdots & \ddots & \vdots \\ K_{hp} & \cdots & K_{hh} \end{bmatrix} \tag{II.45} \]

where \( K_{pp} \) is a \((n_1\times m_1)\)-dimensional matrix, \( K_{ph} \) is a \((n_1\times m_2)\)-dimensional matrix, \( K_{hp} \) is a \((n_2\times m_1)\)-dimensional matrix, and \( K_{hh} \) is a \((n_2\times m_2)\)-dimensional matrix. Then, using Equations (II.28), (II.35), (II.41), (II.44), and (II.45) we obtain:

\[ \hat{x}^+_{p} = \hat{x}^-_{p} + K_{pp}(z - h_p(\hat{x}^-;\alpha,\mu)) + \]

\[ + K_{ph}(z - h_h(\hat{x}^-;\alpha,\mu)) \tag{II.46} \]

and

\[ \hat{x}^+_{h} = \hat{x}^-_{h} + K_{hp}(z - h_p(\hat{x}^-;\alpha,\mu)) + \]

\[ + K_{hh}(z - h_h(\hat{x}^-;\alpha,\mu)) \]
\[ + K_{hh} (\mathbf{z}_h - \mathbf{h}_h (\hat{\mathbf{x}}; \mathbf{s}, \mathbf{u})) \]  \hspace{1cm} (II.47)

It is clear then that feedback coupling is due to non-zero gain matrices: \( K_{ph} \) and \( K_{hp} \). Also, it is noted that forecast errors in precipitation are used to correct hydrologic model states and forecast errors in outflow are used to correct precipitation model states: a two-way coupling.

In order to further examine the nature of the feedback coupling it is necessary to express the matrices \( K_{ph} \) and \( K_{hp} \) as functions of state and noise uncertainty via Equation (II.31). For that purpose partitioning of the covariance matrix \( \mathbf{P} \) is necessary:

\[
\mathbf{P} = \begin{bmatrix}
\mathbf{P}_{pp} & \mathbf{P}_{ph} \\
\cdots & \cdots \\
\mathbf{P}_{hp} & \mathbf{P}_{hh} 
\end{bmatrix}
\]  \hspace{1cm} (II.48)

where \( \mathbf{P}_{pp} \) is a \((n_1 \times n_1)\)-dimensional symmetric covariance matrix, \( \mathbf{P}_{hh} \) is a \((n_2 \times n_2)\)-dimensional symmetric covariance matrix, \( \mathbf{P}_{ph} \) is a \((n_1 \times n_2)\)-dimensional matrix, and \( \mathbf{P}_{hp} \) is a \((n_2 \times n_1)\)-dimensional matrix with:

\[
\mathbf{P}_{ph} = \mathbf{P}_{hp}^T
\]  \hspace{1cm} (II.49)

At this point and for simplification of the derivations involved we set

\[
m_1 = 1
\]  \hspace{1cm} (II.50)

\[
m_2 = 1
\]  \hspace{1cm} (II.51)

Such an assumption is not a binding one in cases of headwater areas without pronounced relief (differences in elevation less than 500 meters), where the two observation variables are mean areal precipitation over the whole basin and outflow discharge (or stage) (see Georgakakos, 1986a-b, and Georgakakos, 1987, for examples). Under assumption-equations (II.50) and (II.51), the inverse in Equation (II.31) is a 2-dimensional matrix.

Partition the observation gradient matrix \( \mathbf{H} \) as follows:
\[ H = \begin{bmatrix} H_{pp} & H_{ph} \\ H_{hp} & H_{hh} \end{bmatrix} \]  \hspace{1cm} (II.52)

where \( H_{pp} \) is a \((1 \times n_1)\)-dimensional matrix, \( H_{ph} \) is a \((1 \times n_2)\)-dimensional matrix, \( H_{hp} \) is a \((1 \times n_1)\)-dimensional matrix, and \( H_{hh} \) is a \((1 \times n_2)\)-dimensional matrix. In addition \( H_{ph} \) and \( H_{hp} \) are matrices whose elements are all equal to zero. Also, partition the \((2 \times 2)\)-dimensional matrix \( R \) as:

\[
R = \begin{bmatrix} R_1 & 0 \\ 0 & R_2 \end{bmatrix} \hspace{1cm} (II.53)

assuming no linear dependence between the errors in observing precipitation and the errors in observing discharge.

The inverse \((2 \times 2)\)-dimensional matrix in Equation (II.31) can then be written as:

\[
(HP^{-T} + R)^{-1} = \begin{bmatrix} R_2 + H_{hh}P_{hh}H_{hh}^T & - & H_{pp}P_{ph}H_{hh}^T \\ H_{hp}P_{hh} & . & . & . & . & . & . & . \\ - & H_{hp}P_{ph}H_{hh}^T & . & . & . & . & . & . \\ R_1 & H_{pp}P_{pp}H_{hh}^T & . & . & . & . & . & . \end{bmatrix} \]  \hspace{1cm} (II.54)

with the scalar determinant \( D \) defined by:

\[
D = R_1R_2 + R_1H_{hh}P_{hh}H_{hh}^T + R_2P_{pp}P_{pp}H_{pp}^T + H_{pp}P_{pp}H_{ph}H_{ph}^T + H_{pp}P_{pp}H_{hh}H_{hh}^T \]  \hspace{1cm} (II.55)

Then, from Equations (II.31) and (II.45) it follows that:

\[
K_{ph} = \begin{bmatrix} P_{ph}H_{ph}R_1 + P_{ph}H_{ph}P_{pp}H_{pp}^T \\ P_{ph}H_{ph}H_{ph}^T + P_{pp}P_{pp}H_{pp}^T \end{bmatrix} \]
\[ - P^T_{pp} P_{hh} P_{hp}^T \] \[ \frac{1}{D} \] (II.56)

and

\[ K_{hp} = \left[ P^T_{hp} P_{hh} R + P_{hp} P_{hh} P_{hp}^T P_{hh}^T \right] \cdot \frac{1}{D} \] (II.57)

The factors that contribute to the initiation of feedback coupling between the meteorological (precipitation) and the hydrological components can be obtained from Equations (II.56) and (II.57) by setting:

\[ P_{ph} = 0 \] (II.58)

where 0 represents an \((n_1 \times n_2)\)-dimensional matrix with all its elements equal to zero. Using Equations (II.58) and (II.49) in Equations (II.56) and (II.57), one obtains:

\[ K_{ph} = 0 \quad \text{and} \quad K_{hp} = 0 \] (II.59)

Thus, coupling is due to the presence of non-zero \(P_{ph}\). Such non-zero \(P_{ph}\) is obtained during the propagation step of the Extended Kalman Filter (Equation (II.27)) because of the mass coupling of the meteorological and hydrological components. That is,

\[ \frac{dP_{ph}}{dt} = F_{pp} P_{ph} + P_{ph} F_{hh}^T + F_{ph} P_{hh} + P_{ph} F_{hp}^T \] (II.60)

assuming that there is no dependence between model errors in the precipitation model and model errors in the hydrologic model, and using:

\[ F_{pp} = \frac{\partial f}{\partial p} \] (II.61)

\[ F_{ph} = \frac{\partial f}{\partial h} \] (II.62)
\[ F_{hh} = \frac{\partial F}{\partial h} h \]  \hspace{1cm} (II.63)

\[ F_{hp} = \frac{\partial F}{\partial p} h \]  \hspace{1cm} (II.64)

Were \( P_{ph} = 0 \) and \( F_{hp} = 0 \), Equation (II.60) would reduce to:

\[ \frac{dP_{ph}}{dt} = F_{pp} P_{ph} + P_{ph} F_{phh} \]  \hspace{1cm} (II.65)

For zero initial matrix \( P_{ph} \), Equation (II.65) yields zero solution for all times. It is then clear that feedback coupling is due to the existence of non-zero \( F_{ph} \) or \( F_{hp} \). State derivative dependence is shown in Equations (II.36) and (II.37). It follows that \( F_{hp} \) is non-zero due to the terms in \( F_{phh} \) that set the input to the hydrologic model equal to the output of the precipitation model.

In summary, mass conservation is responsible for the feedback coupling by the state estimator. An appealing result!

**G. System Configuration**

Using the previously formulated components, an Integrated Hydrometeorological Forecast System (IHFS) was designed for the real time forecasting of floods and flash floods. Figure II.1 presents the IHFS configuration.

In Figure II.1, \( u_1 \) through \( u_N \) represent the input vectors of the meteorological variables from several meteorological sensors located near the drainage basin of interest. Once the meteorological input enters the system, consistency checks are initiated in order to discard erroneous data. At the present stage of development, IHFS includes only checks on the magnitude of the input variables (range quality control). The meteorological data that pass the quality control are interpolated in space in order to arrive at the vector of meteorological variables that characterizes the basin of interest. In cases where mountainous terrain is part of the drainage basin of interest, the basin is divided in several \( L \) orographic zones of increasing altitude and the quality controlled meteorological input is spatially interpolated at the centroid of each orographic zone. The spatial interpolation procedure of Gerogakakos (1986a) that accounts for mountainous terrain is used. The result
of the interpolation is a set of meteorological variable vectors \( \mathbf{u}_{Z_1} \) through \( \mathbf{u}_{Z_L} \) that characterize each one of the \( L \) orographic zones. These vectors constitute the input to the stochastic-dynamic quantitative precipitation forecast (QPF) model which generates mean areal precipitation forecasts QPF\(_1\) through QPF\(_L\) for each orographic zone. A mean areal precipitation forecast QPF\(_B\) for the whole drainage basin is produced as a weighted average of the zonal precipitation forecasts. Each zonal forecast is weighted by the area of the corresponding orographic zone. The drainage basin mean areal precipitation forecast feeds a spatially lumped, hydrologic, quantitative stage forecast (QSF) model (e.g., an API procedure) that: 1) computes and subtracts from the input mean areal precipitation forecast the infiltration losses to the ground, 2) routes the remaining input as overland flow and channel runoff to the point where the flood forecast is needed, and 3) converts the flow discharge to river stage at the point of interest. The output of the hydrologic model is the quantitative stage forecast QSF. Both the basin mean areal precipitation forecast QPF\(_B\) and the stage forecast QSF are IHFS system outputs and they feed the flash-flood warning system (see Figure II.1).

IHFS processes observations of point precipitation from several (K) raingages (if any exist) located within or near the basin, and/or observations of flow stage at the point of interest. At the time that the set of raingage observations \( \mathbf{u}_{R_1} \) through \( \mathbf{u}_{R_K} \) enter the IHFS system and after they pass a range check for validity (range quality controller in Figure II.1), they are divided in categories based on raingage elevation, one category for each orographic zone; and mean areal precipitation estimates for each category are computed: \( \text{MAP}_1 \) through \( \text{MAP}_L \). The observed MAPs are compared to the model produced QPFs that correspond to the same time and orographic zone, and a set of errors \( e_1 \) through \( e_L \) is computed, one error for each orographic zone. Through a similar process, in cases a river stage observation \( u_s \) is available, an error \( e_Q \) is produced by subtraction of the model predicted QSF from the quality controlled observation \( H_{\text{obs}} \) (Figure II.1). The computed errors from all the observations and the predicted state variables of the QPF model \( \hat{x}_P \) and of the QSF model \( \hat{x}_Q \) are input to a state estimator. The estimator 1) updates the predicted state variables based on the computed errors and returns the updated state variables to their respective models so that the next forecasts will be based on updated initial conditions, 2) generates variances for the forecasts QPF\(_B\) and QSF through a set of variance equations and during the forecast stage (no new
observations are yet available), and 3) generates forecasts of the probability that a certain given critical flood stage will be exceeded at various times in the future (namely, the probability of flood occurrence PFO in Figure II,1). The probability PFO is an IHFS system output and it feeds the warning system together with the forecasts QPPB and QSF.

The structure of the IHFS system is capable of accommodating various real-time conditions ranging from sparse data cases to cases with several data sensors and historical datasets (see Georgakakos, et al., 1988; for an exhaustive description of IHFS options). Upper air data from radiosondes can be used when available. However, IHFS can produce forecasts with only surface meteorological observations. The model component that generates surface runoff from rainfall can be a simple Antecedent Precipitation Index (API) model or the conceptual Sacramento soil moisture accounting model (see Georgakakos, 1986a) depending on data availability for calibration. The state estimator is designed in such a way that it can accommodate observation vectors with time varying composition. Such a capability is indispensable for real time implementation due to potential sensor failure during flooding. IHFS can be used both in head-water and in first-order-tributary basins.

H. Real-Time Operation

As it is evident from the previous sections of this chapter, the IHFS design is based on the availability of automated sensors for the observation of meteorological variables such as air temperature, pressure and dew-point temperature (also, wind speed and direction in cases of pronounced relief), precipitation rate, and stage or flow rate. Furthermore for real-time operation, specification of forecasts of the meteorological input variables is necessary. The current version of IHFS uses a simple persistence scheme for the determination of future values of meteorological input variables for the duration of the forecast time interval. One could use regression to improve over the simple persistence. Another possible extension would be to update the design so that IHFS can accept forecasts of meteorological variables produced by the large-scale numerical weather prediction models which run routinely at the National Meteorological Center.

Under the present design, the long-lead time forecasts (more than one time-step lead times) depend on the duration specified for the persistence of
the storm conditions over the drainage basin of interest. For that duration the simple persistence scheme (which forecasts current observations) is used to supply the forecast meteorological input to the hydrometeorological models of IHFS. Beyond that duration, fair weather is assumed. Operational use of IHFS would, therefore, be facilitated if observations from remote sensors such as weather radars and satellites were used to assess the storm extent and motion characteristics and, consequently, its duration over the basin of interest. Such observations can also help specify the most appropriate meteorological station for meteorological input data.

Previous studies (Georgakakos, 1987) have shown that when both precipitation and stage (or discharge) data is available and error-free, IHFS performance is very robust. However, under operational conditions such data may not be readily available. It is then imperative to examine the sensitivity of IHFS predictions to errors in the data. Those errors may be due to a malfunction or failure of equipment, or they may be due to the preprocessing of data. For example, they may be due to the computation of mean areal precipitation from several raingage observations, or the computation of discharge from stage recordings and rating curves. Such a sensitivity analysis also examines the sensitivity with respect to the various data types that IHFS uses. As a result, inference regarding the importance of various data groups to real time flood forecasting can be made. The next Chapter examines the sensitivity of IHFS predictions to data errors and establishes guidelines for real time operation.
III. CASE STUDIES

A. Description of Test Drainage Basins and Data

Data from a total of three drainage basins of varying hydromorphological characteristics were used in the sensitivity analysis. The reason for using more than one basin was to examine the sensitivity of IHFS predictions to basin-dependent variables and to various data configurations. Table III.1 presents the important characteristics of each of the drainage basins for easy reference. Selected events were used from the records of each basin ranging from a flash flood with a response time of about 50 minutes to a flood event in a small basin of mild slope with a response time of more than a day. The following sections present the drainage basins, their observations, and the event-data used in the analysis.

A.1. North Fork Big Thompson River at Drake, Colorado

The North Fork of Big Thompson River, with outlet near Drake, Colorado, is an 85.1 square mile (218 km²) headwater basin located on a mountain barrier to easterly airflow (hatched area in Figure III.1). The North Fork is one of the two main streams that feed the Big Thompson River. Elevation in the catchment ranges from about 6200 feet (1889.76 m) near Drake to about 10,600 feet (3230.88 m) on the upper parts of the catchment. Pronounced orographic effects on precipitation are observed in a convective environment with strong winds blowing from an easterly direction.

The catchment response time is less than 1 hour, classifying this case at the lower end of flash flood prone catchments in terms of response time.

This catchment is known for the destructive flash flood of July 31 to August 1, 1976, that claimed more than 200 lives in the general Big Thompson River area. (The source for information on this event was USGS (1979)).

Figure III.1 presents the predominant surface wind velocity direction and magnitude for several locations in the area during the flash flood event. It can be seen that for stations sufficiently removed from the mountain barrier, the wind is blowing at an angle almost perpendicular to the mountain range. The station at Denver's Stapleton Airport was used to provide upwind hourly surface meteorological data (temperature, pressure, dew-point temperature,
<table>
<thead>
<tr>
<th></th>
<th>Big Thompson at Drake</th>
<th>Tug Fork at Iaeger</th>
<th>West Otter Creek at Kanawha</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Area (km²):</strong></td>
<td>218</td>
<td>1285</td>
<td>47</td>
</tr>
<tr>
<td><strong>Latitude:</strong></td>
<td>40 23'</td>
<td>37 28'</td>
<td>42 57'</td>
</tr>
<tr>
<td><strong>Longitude:</strong></td>
<td>105 21'</td>
<td>81 55'</td>
<td>93 47'</td>
</tr>
<tr>
<td><strong>Highest Elevation (m):</strong></td>
<td>3230</td>
<td>790</td>
<td>380</td>
</tr>
<tr>
<td><strong>Lowest Elevation (m):</strong></td>
<td>1890</td>
<td>360</td>
<td>355</td>
</tr>
<tr>
<td><strong>Response Time (hrs):</strong></td>
<td>1</td>
<td>12</td>
<td>30</td>
</tr>
<tr>
<td><strong>Land Use:</strong></td>
<td>Forested</td>
<td>Forested</td>
<td>Cultivated Fields</td>
</tr>
</tbody>
</table>
Figure III.1. North Fork Big Thompson River basin and prevailing surface wind.
wind direction, wind magnitude). Hourly precipitation data from one rain gauge located at Glen Haven (Figure III.1) at an altitude of 7,200 feet (2194.56 m) was used. Hourly stage data at Drake, Colorado, were also utilized.

Figure III.2 presents the heart of the catchment of North Fork, Big Thompson River, with the elevations of several locations in the catchment shown. Also shown in Figure III.2 are estimated (USGS, 1979) isohyets of storm total precipitation. A maximum of about 10 inches (25.4 cm) was estimated as a storm total peak. Under the steep terrain and excessive rainfall conditions of the flood, no controls were imposed by the soil to the flood flows. It is noted that the location of the rain gauge (at Glen Haven) was such that a good estimate of the heavier precipitation was obtained in real time.

The catchment was divided into two orographic zones with area-weighted average elevations of 7,500 feet (2286 m) and 6,600 feet (2011.68 m). For the computation of probabilistic flood occurrence forecasts the flood stage was set equal to 5.7 ft (1.74 m).

Figure III.3 presents the observed time series of all the input and output variables processed by IHFS (simulating real time conditions) during the sensitivity analysis of the flood event. The source of the data was USGS (1979).

A.2. Tug Fork at Iaeger, West Virginia

The 502 square mile (1285 km²) Tug Fork catchment with outlet at Iaeger, West Virginia, is located at the southern border of West Virginia with the state of Virginia (Figure III.4). The catchment experiences frequent flooding, classified by the National Weather Service Forecast Office at Charleston, West Virginia, as flash flooding, with significant damage downstream. With a response time of about 12 hours, this catchment is at the upper end of flash flood prone catchments in terms of response time. The closest meteorological station is located at Beckley, West Virginia, at a distance of about 40 miles (64 km) toward the north-northeast. Precipitation data is collected by automated sensors that are part of the U.S. National Weather Service Eastern Region's IFFLOWS system. Figure III.5 presents the location of the precipitation-recording stations in and around the catchment. Also shown in Figure
Figure III.2. Spatial distribution of the July-August, 1976 storm-total rainfall over the North Fork Big Thompson River basin
Figure III.3. Observed time series of IHFS input and output variables for the July-August, 1976 flash flood of Big Thompson River
Figure III.3. Observed time series of IHFS input and output variables for the July-August, 1976 flash flood of Big Thompson River (Continued).
Figure III.4. Geographical location of the Tug Fork basin with outlet at Iaeger, West Virginia
Figure III.5. The Tug Fork basin and automated precipitation recording stations
III.5 is the elevation in feet of several locations within the catchment. The area is located within a mountainous region which makes for high stream velocities. No significant orographic effects could be identified for the catchment, and this is attributed to the fact that it does not lie on a significant mountain barrier. One orographic zone was used with the IHFS.

On May 8, 1984 the area experienced heavy rainfall, followed by significant flooding. The center of the storm was located to the northwest of the catchment. The IHFS was used for this case, with 3-hourly data of surface air temperature, dew-point temperature, pressure, precipitation, and stage at Iaeger. The forecast lead time was set equal to the observation interval time (that is, 3 hours). For the sensitivity analysis of the spatial interpolation procedure for the meteorological surface data (developed by Georgakakos, 1986a), the meteorological station located at Huntington, West Virginia, was used, in addition to the station at Beckley, West Virginia (see Figure III.4). This was done because of the proximity of the former station to the storm center. For the computation of probabilistic flood occurrence forecasts the flood stage was set equal to 20 ft (6.1 m).

Figure III.6 presents the observed time series of all the input and output variables processed by IHFS during the sensitivity analysis of the flood event. The data was provided by the staff of the National Weather Service Forecast Office at Charleston, West Virginia.

A.3. West Otter Creek at Kanawha, Iowa

The drainage basin of West Otter Creek with outlet near Kanawha, Iowa, is an 18.2 square-miles (47 km²) headwater basin that feeds into the Boone River which is a tributary of the Des Moines River. The basin is located in north-central Iowa at an average elevation of about 370 m above mean sea level. The highest elevation is about 380 m and the lowest elevation is about 355 m above mean sea level. The basin response time is about 30 hours due to generally mild slopes. Most of the land area is cultivated and it consists mainly of corn and bean fields.

Figure III.7 presents the catchment area and indicates the location of the automated meteorological station which records air temperature, pressure, dew-point temperature, and precipitation. The location of the automated stage-recording station is also shown. The automated observation stations
Figure III.6. Observed time series of IHFS input and output variables for the May, 1984 flood of Tug Fork
Figure III.6. Observed time series of IHFS input and output variables for the May, 1984 flood of Tug Fork (Continued).
Figure III.6. Observed time series of IHFS input and output variables for the May, 1984 flood of Tug Fork (Continued).
Figure III.7. The West Otter Creek basin with outlet at Kanawha, Iowa, and the automated recording station
have been in operation since the beginning of 1987 as part of the U. S. Army Corps of Engineers Inland Waterways Demonstration Project. Data from the automated recording stations are routinely collected and stored in real time in the database of the U.S. Army Corps of Engineers Rock Island District. There were two significant storm events (one in April 1988 and one in September 1987) since the observation stations begun operating. The September 1987 event data contained several missing values of relative humidity which made its inclusion in the sensitivity analysis impossible. Hourly data from the April 1988 were used in the sensitivity analysis. For the computation of probabilistic flood occurrence forecasts the flood stage was set equal to 2.8 ft (0.84 m).

Figure III.8 presents the observed time series of all the input and output variables processed by IHFS during the sensitivity analysis of the West Otter Creek flood event. It can be seen that eventhough the catchment area is less than 19 square miles (and no longer than 6.5 miles), the time interval between the time of the volume centroid of hyetograph and the time of the volume centroid of the hydrograph is longer than 30 hours. Such long response time is attributed to the low slope of the catchment and the active role of sub-surface flow. In fact, volume analysis of the hyetograph and hydrograph (using any of the stage-discharge rating curves presented below and assuming that the single raingage readings are representative of the mean areal precipitation over the basin) of the April 1988 flood indicates that considerable volume of water might be leaving the basin through routes different from the West Otter Creek route (especially in view of the fact that sub-surface water collection-tiles exist within the basin).

Since the available drainage-basin outflow data is in the form of stage recordings and the channel routing model of Section II.D produces discharge predictions (see equation (III.15), a rating table is needed to convert discharge to stage and vice-versa. Figure III.9 presents the observed pairs of stage and discharge at the outlet of West Otter Creek near Kanawha. It is noted that the creek has been channelized at that location and that the observed pairs have been measured by two different observer groups (see Figure III.9). Due to the dry weather that has prevailed since the stage recorder begun operating, low values of stage and discharge have been observed. In fact, no observed values exist for pairs with stages greater than 2.7 ft.
Figure III.8. Observed time series of IHFS input and output variables for the April, 1988 flood of West Otter Creek.
Figure III.8. Observed time series of IHFS input and output variables for the April, 1988 flood of West Otter Creek (Continued).
Figure III.9. Observed stage-discharge pairs at the proximity of the stage recording station of West Otter Creek
Therefore, in order to perform sensitivity analysis with data from the flood event of April 1988, extrapolation of the rating curve in the region of high stages is necessary. Four extrapolation methods were used in an effort to examine the sensitivity of IHFS stage predictions to the rating curve errors:

1. Linear extrapolation of the linear segment between the highest two pairs of Figure III.9.
2. A third-order polynomial fit of the values of the stage-discharge pairs transformed in the space of natural logarithms (Figure III.10).
3. A linear extrapolation of the linear segment that connects the lowest and the third lowest pair of Figure III.9 (Figure III.11).
4. Uniform flow analysis with Manning's roughness formula assuming a rectangular cross-section with a top width of 10 ft, a value of Manning's n equal to 0.020 (which corresponds to a lined canal) and a bottom slope of 1 ft/mile (Figure III.12).

B. Performance Measures

IHFS produces predictions of both the mean value of stage at each forecast time but also of the associated prediction error standard deviation. Furthermore, by assuming that the prediction errors are normally distributed for each forecast time, the system also generates the probability that a certain threshold of stage will be equaled or exceeded at each forecast time (see Section II.E, Equation (II.34)).

The performance measures used in the sensitivity analysis were functions of: a) the predicted mean value of stage at each forecast time, and b) the probability of flooding at each forecast time. These errors are relevant to the prediction of the timing and magnitude of flood stages under uncertain input as well as to the prediction of hydrograph volume (relevant to the uses of IHFS predictions as input to reservoir control schemes). The probability errors were used in the performance measures because of the importance of probabilistic predictions to modern day decision making regarding real-time reservoir control (Georgakakos, A. and Marks, 1987). Graphical displays of the IHFS predictions were produced that depict the two aforementioned forecast quantities for all the sensitivity analysis cases and for each drainage basin.
Figure III.10. Third-order polynomial fit to the observed values of stage-discharge pairs for West Otter Creek
Figure III.11. Linear extrapolation using low-valued stage-discharge pairs for West Otter Creek
Figure III.12. West Otter Creek rating curve based on uniform flow analysis using Manning's formula with $n = 0.02$, $w = 10$ ft, $S = 1$ ft/mi
C. Sensitivity Analysis Design

C.1. A Framework for Sensitivity Analysis

Examination of the sensitivity of a scalar function \( P(\mathbf{x}) \) to changes in the values of the elements of the vector \( \mathbf{x} \) consists of evaluating the derivatives:

\[
S_j(\mathbf{x}) = \frac{\partial P(\mathbf{x})}{\partial x_j}; \ j = 1, 2, \ldots, N
\]  

(III.1)

where \( N \) is the dimension of the vector \( \mathbf{x} \). Specific values \( S_j(\mathbf{x}_i) \) \((i=1,2,\ldots,p)\) can be determined in a local sensitivity analysis for certain values \( \mathbf{x}_i \) which are set by the objectives of the analysis. For example, assume that \( P(\mathbf{x}) \) represents annual savings obtained by reservoir flood control and \( \mathbf{x} \) is the set of variables that define a release policy for the stored water. \( S_j \) can be evaluated for specific release policies \( \mathbf{x}_i \): a conservative release policy, a long-term release policy, etc.

A particular case arises when \( \mathbf{x} \) is a function of time \( t \). In such cases \( S_j \) \((j=1,2,\ldots,N)\) is a function of \( t \) and a choice should be made regarding the times \( t_i \)(\(i=1,2,\ldots,p\)) for which \( \mathbf{x}(t) \) will be evaluated, so that \( \mathbf{x}_i = \mathbf{x}(t_i) \). Then,

\[
S_j(\mathbf{x}(t_i)) = \frac{\partial P(\mathbf{x}(t_i))}{\partial x_j}; \ j = 1, 2, \ldots, N; \ i=1,2,\ldots,p
\]  

(III.2)

When \( S_j \) cannot be determined analytically by the previous expression, the numerical approximation is used:

\[
S_j(\mathbf{x}(t_i)) = \frac{\Delta P(\mathbf{x}(t_i))}{\Delta x_j(t_i)}; \ j=1,2,\ldots,N; \ i=1,2,\ldots,p
\]  

(III.3)

where \( \Delta \) denotes the difference operator with \( \Delta x_j \) representing a small increment. The issue of selecting \( \Delta x_j \) arises because of the possibility of having a different value of \( S_j \) for different values of the increment \( \Delta x_j \). A range of values of \( \Delta x_j \) can be used and a graph of \( S_j \) as a function of \( \Delta x_j \) can serve as a basis for defining a range of values of \( \Delta x_j \) for which the value of \( S_j \) remains stable.
C.2. Sensitivity Analysis of the IHFS Forecasts

The previous formulation for sensitivity analysis is now used to examine the sensitivity of the IHFS forecasts to changes in the value of various quantities. The function $\Delta P$ is defined as the absolute value of the maximum difference between the nominal stage prediction of IHFS and the stage predictions corresponding to the IHFS using different from nominal values for selected quantities. In this context nominal refers to the calibrated IHFS using as input the observed values of input variables, and as observations of precipitation and stage (for updating) the actual observations of these quantities.

In mathematical terms,

$$P(\bar{x}(t_1)) = \text{ABS} \left[ \max_{\Delta} \left\{ P(t_k (x(t_1) + \Delta x(t_1)) - P(t_k (x(t_1))) \right\} \right]$$

$$t_{i+1} \leq t_k \leq t_{i+v}$$

$$i = 1, 2, \ldots, p$$

(III.4)

where $\text{ABS}[\cdot]$ denotes the absolute value function, $\max \{\cdot\}$ denotes the maximum value function, $\Delta x(t_1)$ is the vector whose elements are equal to zero apart from the $j$th element which is equal to $\Delta x_j(t_1)$, $t_k$ is the $k$-th forecast time, $t_{i+v}$ is the last forecast time with:

$$t_i < t_{i+1} < \ldots < t_k < \ldots < t_{i+v}$$

In Equation (III.4) $\bar{x}(t_1)$ corresponds to the nominal IHFS operation.

As formulated in Equation (III.3), sensitivity is defined with respect to isolated small changes (at times $t_1$) in the values of the quantities of interest $x_j(t_1), j = 1, 2, \ldots, N$. It is possible to examine the IHFS sensitivity to changes in the value of $x_j$ that persist over a period of discrete time: $[t_1, t_{i+\ell}]$. Then, the sensitivity index $S_j (j=1, 2, \ldots, N)$ becomes a function of all: $\bar{x}(t_1), \bar{x}(t_{i+1}), \ldots, \bar{x}(t_{i+\ell})$. Define vectors $\bar{y}_k^i$ and $\bar{y}_k$ such that

$$\bar{x}(t_1) + \Delta x(t_1)$$

$$\bar{y}_k^i = \frac{\bar{x}(t_k) + \Delta x(t_{i+k})}{\bar{x}(t_{i+k})} \quad ; \quad k = 1, 2, \ldots, \ell$$

(III.5)

50
and

\[ X(t_i) \]
\[ Y_k = \quad ; \quad k = 1, 2, \ldots, \ell \quad (III.6) \]

\[ X(t_{i+k}) \]

with \( t_i < t_k \). Then, for the case of persistent changes, \( \Delta P \) can defined as:

\[ \Delta P(Y_k) = \frac{1}{v} \sum P_{t_i+1} (Y_i') - P_{t_i+1} (Y_i); \ldots ; \]
\[ P_{t_i+k+1} (Y_i+k) - P_{t_i+k+1} (Y_i+k); \ldots; P_{t_i+k+1} (Y_i+k) - P_{t_i+k+1} (Y_i+k); \ldots; \]
\[ P_{t_i+v} (Y_i+v) - P_{t_i+v} (Y_i+v) \]  

(III.7)

where \( \sum \) represents the sum of terms separated by semi-colon and \( v \) is the lag for which prediction changes occur.

It is noted that the previous sensitivity formulation can also be used to study the sensitivity of IHFS forecasts to isolated or persistent errors in input and output (for updating options) data.

An effort was made to use as many as possible of the IHFS options that could normally be activated in operational use of the system. Thus, all the three cases of no real time state updating, real time updating from mean areal precipitation, and real time updating from both mean areal precipitation and stage, were examined. The three cases could correspond to various data-availability scenarios during operational use. Thus, when no raingages and stage gages are available or when the corresponding available sensors are malfunctioning the no-state updating option would be used. In other cases when no valid stage data is available state updating from only precipitation would be used. The case of updating from only stage was not examined during the sensitivity analysis (even though IHFS is capable of this option) because it was considered improbable that stage data would be available in practice without attendant precipitation observations from raingage sensors in the area.

One of the basins (the North Fork Big Thompson River at Drake, Colorado) is on a mountain barrier which produces significant orographic enhancement of precipitation under certain airflow conditions. Thus, it presented the opportunity to examine the sensitivity of stage forecasts to input data with and without the IHFS orographic enhancement option. Such analysis was carried out and is presented in a later section.
The effect of the location of the station providing the meteorological input (with respect to the area of storm moisture inflow) on the sensitivity of the IHFS predictions to input data was also examined for the case of the Tug Fork at Iaeger, West Virginia, drainage basin. Such an analysis is important when the input meteorological data are recorded at stations located at a considerable distance from the drainage basin of interest.

The sensitivity of the IHFS stage predictions to the parameters of the API runoff-generating component used was reported in Georgakakos (1987) for the drainage basins of the North Fork Big Thompson River and Tug Fork. A similar analysis is presented in a later section for the drainage basin of West Otter Creek. These analyses show that there are no significant effects of the various sets of API parameters used on the stage predictions. Thus, an arbitrary choice of the values of the API parameters was made for each drainage basin, and these values were used during all the sensitivity runs of IHFS.

With respect to the type of IHFS data changes or errors introduced during the sensitivity analysis (see Section C.2., Equations (III.3), (III.4) and (III.7), both isolated and persistent changes or errors were examined. As previously discussed, an isolated error in an input variable is an error that occurs at one time step and it simulates a single data entry error which may be due to a temporary malfunction of the sensor recording the input variable or due to an error in manual coding of the data. We introduced isolated input-data errors at two different times \( p=2 \) for each case in order to examine the sensitivity of IHFS predictions to the timing of error (e.g., at the beginning of the rising limb of a hydrograph and near the peak of the hydrograph). A persistent error is an error due to a failure of a sensor that generates and possibly transmits recordings that are erroneous. It is noted that the errors introduced during the sensitivity analysis are not large enough to be discarded by the IHFS range quality controller (see Figure II.1). In fact the results of the present sensitivity analysis could be used to establish guidelines for the design of more effective IHFS input-data quality controllers.

Isolated changes or errors in the meteorological variables were examined for all three drainage basins. In particular errors in the dew-point temperature of magnitudes \( +2^\circ, -2^\circ, +5^\circ, -5^\circ \), were introduced at the beginning and near the peak of the hydrograph. For the West Otter Creek basin persistent
errors in the relative humidity readings were introduced during the storm that caused the flood event under study. Isolated or transient errors in the dew point (or relative humidity) affect the precipitation prediction component of IHFS since they modify the condensed water mass. Thus, analysis of the sensitivity of IHFS predictions to such errors is equivalent to the sensitivity of the stage predictions to the mean areal precipitation predicted by the precipitation component.

Isolated changes or errors in the mean areal precipitation were introduced by altering the data values of one or more raingages which carried non-zero weights in the computation of mean areal precipitation. Thus, in the case of Tug Fork (with a mean areal precipitation computed from data recorded at each of 8 automated raingage stations) and for designated times, errors were introduced to the highest, highest and second highest, lowest, lowest and second lowest raingage data values. For the other two basins there was only one automated raingage station available and thus errors in the data values of this station were introduced at designated times. For the West Otter Creek basin a persistent error was introduced to simulate failure of the single automated gage. The specifics of the error magnitude and the times of errors are detailed in later sections for each basin. In general, the precipitation input errors ranged in magnitude from -50% to +100% of the recorded values at the corresponding times.

Stage changes or errors were introduced at two different times for each of the three drainage basins: near the beginning of the rising limb of the hydrograph and near the peak of the hydrograph. The errors ranged in magnitude from -50% to 100% of the corresponding stage data values.

In the following sections we present the results of the sensitivity analysis for each drainage basin. A schematic options tree has been constructed for the sensitivity analysis of each basin to guide the reader amongst the numerous plots. A code that characterizes the set of options that were active during the IHFS runs that created the results depicted in each sensitivity analysis plot was also defined for convenience. Table III.2 presents the key to the code.
Table III.2

Key to Code Characterizing Sensitivity Analysis Option-Sets

**Code Fields**: A, B, C

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**Code Field Description**

- **A**: Variable(s) whose data have been used for real-time state updating.
  - **Possible Values**:
    - P: Precipitation.
    - PH: Precipitation and stage.
    - O: No state updating is used.

- **B**: Variable whose data have been modified to include changes or errors for sensitivity analysis.
  - **Possible Values**:
    - TD: Surface air dew-point temperature.
    - X: Rainfall
    - H: Stage

- **C**: Indicator of the station supplying the meteorological data. This description applies only to the Tug Fork flood when sensitivity with respect to the meteorological station location is performed.
  - **Possible Values**:
    - 1: First meteorological station (Beckley, WV).
    - 2: Second meteorological station (Huntington, WV).
    - 1 and 2: First and second meteorological stations.

- **C**: Indicates whether orographic enhancement computations have been performed. This description applies only to the North Fork Big Thompson River flood.
  - **Possible Values**:
    - OR: Orographic rainfall-enhancement computations have been performed.
    - O: No orographic enhancement of rainfall was computed.

- **C**: Symbol(s) to indicate the manner in which errors have been introduced in the raingage observations. This description applies only to the Tug Fork flood when sensitivity with respect to rainfall input is performed.
  - **Possible Values**:
L1 : Error introduced in the data of the raingage with the current lowest recorded value.

L1L2 : Errors introduced in the data of both the raingages with the current lowest and second lowest recorded values.

H1 : Error introduced in the data of the raingage with the current highest recorded value.

H1H2 : Errors introduced in the data of both the raingages with the current lowest and second lowest recorded values.
D. Results for the Big Thompson River Flash Flood

The 1976 North Fork Big Thompson River flash flood offered the opportunity to perform sensitivity analysis of the IHFS hourly stage predictions and probability of flooding predictions to changes (or errors) in the input variables under several IHFS option sets. The schematic tree of option sets is presented in Figure III.13. The codes of Table III.2 have been used. For each of the paths of Figure III.13 two plots of predictions vs. observations of stage and two plots of probability of flooding predictions were constructed. Each of the two plots corresponds to the introduction of isolated changes at each of two different times during the flash flood. The times of changes (or errors) are indicated on the figures. Appendix A presents all the relevant plots. Section A.1 presents results corresponding to the mean stage predictions of IHFS, while Section A.2 presents analogous results corresponding to the probability of flooding predictions of IHFS.

The first observation that can be made after a comparison of the Figures in Section A.1 with those in Section A.2 is that the results that correspond to stage predictions are qualitatively similar to the results that correspond to probability of flooding predictions. Predictions of stage have been computed based on Equation (II.32) while predictions of probability of flooding have been computed based on Equation (II.34). It is noted that the stage predictions depend only on the predicted state mean while the probability of flooding predictions depend both on the predicted state mean and covariance (through Equation (II.33)). Given that input changes were only introduced in the mean value of inputs, such qualitative agreement between predictions of stage and probability of flooding is not surprising. Therefore, in the following we will only refer to the stage predictions implying that analogous conclusions can be drawn for the probability of flooding predictions.

The effect that changes in dew-point temperature, precipitation and stage have on stage predictions is more pronounced for high flow periods as compared to low flow periods (e.g., compare Figures A.1.1 and A.1.2, Figures A.1.3 and A.1.4, Figures A.1.9, and A.1.10, and Figures A.1.15 and A.1.16). It is evident from the figures that the time of introduction of an input change (or error) is very important in defining the effects of the introduced change on the predictions. If input changes are introduced at times near the peak of the observed stage-hydrograph, significant prediction sensitivity is observed.
Figure III.13. Sensitivity analysis tree for the North Fork Big Thompson River flash flood
in all cases. If input changes are introduced early in the rising limb of the hydrograph, IHFS predictions do not show significant sensitivity. This observation carries an important implication for the quality control problem of IHFS input data. That is, data quality control should be time dependent. Data quality control is particularly important for high flows. The changes in precipitation and stage introduced were in the form of a percent of observed flow change. It follows then that the resultant prediction deviations depended on the absolute magnitude of the introduced change.

Significant bias results in cases without orographic enhancement of rainfall (e.g., compare Figures A.1.1 and A.1.2 with Figures A.1.3 and A.1.4). It is noted that the bias is equivalent to the bias that either a 5 degree increase in dew-point temperature or a 100 percent increase in stage near the peak of the stage hydrograph produce (see Figures A.1.4 and A.1.12).

The effects of the introduction of a change in input data are more pronounced for the time step immediately following the time of introduction of the change and they are progressively smoothed for subsequent time steps. Faster smoothing occurs when updating from both precipitation and stage is in effect. For this catchment the effects are practically non-existent after three time steps following the time-step when the input data change was introduced (e.g., Figures A.1.1, A.1.2, A.1.13, and A.1.21).

For all cases examined for North Fork Big Thompson River, rainfall data changes produced the less significant effects on IHFS stage forecasts. For this catchment, changes in meteorological data produced the most significant effects overall. For the cases of updating from both precipitation and stage and for changes introduced in the stage data (see Figures A.1.10 and A.2.10), the IHFS stage and probability of flooding predictions are more sensitive to negative changes than they are to positive changes.

Finally, the magnitude of the maximum errors in each case is depicted in Table III.3 when updating from both rainfall and stage data is active and when orographic enhancement of rainfall is included. This Table presents the sensitivities as defined by Equations (III.3) and (III.4). The options chosen are the ones that would be used in practice given the dataset available for this flash flood. For illustration purposes, Figures A.1.2, A.1.6, and A.1.10 (which correspond to the stage results in Table III.3) have been reproduced as Figures III.14, III.15, and III.16, respectively.
Table III.3
Sensitivity of Stage and Probability of Flooding Predictions to Input Data for the North Fork Big Thompson River Flash Flood
(Updating from both Rainfall and Stage)

<table>
<thead>
<tr>
<th>Case</th>
<th>Stage</th>
<th>Probability of Flooding</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dew-point</td>
<td>0.72 ft/degree</td>
<td>0.075 degree$^{-1}$</td>
</tr>
<tr>
<td></td>
<td>(8.8% per degree)</td>
<td></td>
</tr>
<tr>
<td>Rainfall</td>
<td>0.09 ft/inch</td>
<td>0.007 inch$^{-1}$</td>
</tr>
<tr>
<td></td>
<td>(1.0% per inch)</td>
<td></td>
</tr>
<tr>
<td>Stage</td>
<td>0.55 ft/foot</td>
<td>0.040 foot$^{-1}$</td>
</tr>
<tr>
<td></td>
<td>(5.9% per foot)</td>
<td></td>
</tr>
</tbody>
</table>
Figure III.14. IHFS stage predictions for an isolated change in dew-point temperature taking on several values vs. stage observations. Updating from both rainfall and stage is active. Big Thompson River.
Figure III.15. IHFS stage predictions for an isolated change in rainfall taking on several values vs. stage observations. Updating from both rainfall and stage is active. Big Thompson River.
Figure III.16. IHFS stage predictions for an isolated change in stage taking on several values vs. stage observations. Updating from both rainfall and stage is active. Big Thompson River.
E. Results for the Tug Fork Flood

Several cases of sensitivity analysis with respect to changes in input data were performed for the Tug Fork flood of 1984. Three-hourly stage predictions were produced by the IHFS simulating real-time operation. The sensitivity analysis tree is presented in Figure III.17. The codes of Table III.2 have been used. For each of the paths of Figure III.17 two plots of predictions vs. observations of stage and two plots of probability of flooding predictions were constructed. Each of the two plots corresponds to the introduction of isolated changes at each of two different times during the flash flood. The times of changes (or errors) are indicated on the figures. Appendix B presents all the relevant plots. Section B.1 presents results corresponding to the mean stage predictions of IHFS, while Section B.2 presents analogous results corresponding to the probability of flooding predictions of IHFS. As previously noted in Section III.D for the Big Thompson River flash flood, the two types of results are similar for the reason that only changes in the mean of the input data were introduced.

A general comment on the results of Appendix B is that the case of updating from both rainfall and stage data had a much better performance than the case of updating from only rainfall data and the case of no updating. Significant bias is developed for the high flow predictions when updating from only rainfall data is active. An even larger bias develops for the high flow predictions when there is no updating (e.g., compare Figures B.1.1 and B.1.2 with Figures B.1.17 and B.1.18 and with Figures B.1.31 and B.1.32). It is conceivable that the values of the parameters of the soil model used could be improved to have better predictions of the deterministic model. Nevertheless, the sensitivity analysis indicates that for floods such as the one under consideration here updating from both rainfall and stage leads to robust IHFS performance.

In contrast to the previous flash flood examined, the Tug Fork flood did not exhibit significant differences in sensitivity with respect to the timing of the introduced changes (e.g., compare Figures B.1.1 and B.1.2, Figures B.1.13 and B.1.14, and Figures B.1.15 and B.1.16). Similarly, no significant differences in the sensitivity results can be seen when different meteorological stations provided the input air temperature, dew-point temperature and pressure (e.g., see Figures B.1.1 through B.1.6, Figures B.1.17 through
Figure III.17. Sensitivity analysis tree for the Tug Fork flood
B.1.22, and Figures B.1.31 through B.1.36). Finally, no significant sensitivity exists to changes in the rainfall data; both in the highest and in the highest and second-highest values, and in the lowest and in the lowest and second-lowest values (e.g., see Figures B.1.7 through B.1.14, and Figures B.1.23 through B.1.30). High sensitivity of stage and probability of flooding predictions to changes in input stage values is evident (e.g., Figures B.1.15 and B.1.16). The effects of changes in input stage values are most pronounced for the time step following the time step when the change was introduced. The effects are rapidly approaching zero after two time steps from the time step of stage-change introduction (see Figures III.18 and III.19 which are Figures B.1.15 and B.1.16 reproduced here for easy reference).

Finally, the magnitude of the maximum errors in each case is depicted in Table III.4 when updating from both rainfall and stage data is active. The sensitivities were computed based on Equations (III.3) and (III.4). The options chosen are the ones that would be used in an actual IHFS application given the dataset available for this flood.

**F. Results for the West Otter Creek Flood**

For the West Otter Creek flood of April 1988 in addition to the sensitivity analysis with respect to errors in input data, several other analyses were performed. They were motivated by the lack of many sets of historical data for the calibration of critical IHFS input parameters such as the rating curve values of stage and discharge for high flows and the response time of the drainage basin. Due to the increased uncertainty in the West Otter Creek case study and due to the very long response time of the basin (which is greater than a day) with respect to the time interval of observed data (which is equal to 1 hour) it was expected that the state updating component would be the most sensitive component of IHFS. Therefore all the sensitivity studies were performed with updating from both rainfall and stage data being active. All the runs were performed on the HARRIS main frame computer of the U. S. Army Corps of Engineers Rock Island District, using data from the real-time hydrometeorological database residing on the HARRIS. Appendix C contains all the relevant plots. Section C.1 presents results corresponding to the mean stage predictions of IHFS, while Section C.2 presents analogous results corresponding to the probability of flooding predictions of IHFS.
Figure III.18. IHFS stage predictions for an isolated change in stage taking on several values vs. stage observations. Updating from both rainfall and stage is active. Tug Fork. Change time: 1:00pm, May 6, 1985.
Figure III.19. IHFS stage predictions for an isolated change is stage taking on several values vs. stage observations. Updating from both rainfall and stage is active. Tug Fork. Change time: 1:00 a.m., May 7, 1985.
Table III.4

Sensitivity of Stage and Probability of Flooding Predictions to Input Data for the Tug Fork Flood
(Updating from both Rainfall and Stage)

<table>
<thead>
<tr>
<th>Case</th>
<th>Stage</th>
<th>Probability of Flooding</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dew-Point</td>
<td>0.06 ft/degree</td>
<td>0.005 degree⁻¹</td>
</tr>
<tr>
<td></td>
<td>(0.30% per degree)</td>
<td></td>
</tr>
<tr>
<td>Rainfall</td>
<td>0.63 ft/inch</td>
<td>0.036 inch⁻¹</td>
</tr>
<tr>
<td></td>
<td>(4.2% per inch)</td>
<td></td>
</tr>
<tr>
<td>Stage</td>
<td>0.83 ft/foot</td>
<td>0.026 foot⁻¹</td>
</tr>
<tr>
<td></td>
<td>(8.7% per foot)</td>
<td></td>
</tr>
</tbody>
</table>
Figure III.20 (also Figure C.1.1) shows the sensitivity of the IHFS stage predictions to changes in the values of the API soil-model parameters. The parameter sets used were as reported by Georgakakos (1987) based on Betson, et al. (1969). All the available data were used (updating from both precipitation and stage was active) simulating real-time conditions. It can be seen that for a wide range of parameter values the real-time IHFS predictions are not significantly sensitive to the values of the soil moisture accounting model parameters. This results agree with the results of Georgakakos (1987) obtained for the other two case studies of this report. Similar results were obtained for probability of flooding predictions (see Figure C.2.1).

Figure III.21 (also Figure C.1.2) presents the results of a sensitivity analysis with respect to the method of rating curve extrapolation (see Section III.A.3 for a discussion on rating curves). A response time of 35 hours was used and an arbitrary choice was made for the set of API-model parameters. Of significance are the results corresponding to the time interval from 26 April 1988 through 29 April 1988 where a double peaked stage hydrograph was observed. Assuming that the recordings of the stage gage for high flows are free of errors, all the predictions succeed in capturing the hydrograph shape and peak. It is obvious from an inspection of Figure III.21 that the low flow extrapolation method and the uniform flow method for rating curve definition have the greatest errors of the four methods with the other two methods having about the same performance. Particularly successful in predicting the timing and magnitude of both peaks was the use of the third order polynomial method for the extrapolation of the rating curve. Similar trends are observed in Figure C.2.2 corresponding to probability of flooding predictions. Figure III.22 (also Figure C.1.3) presents analogous results for stage prediction and, for a response time of 45 hours. Comments similar to those made above can also be made for this group of predictions. In comparing Figures III.21 and III.22 one can state that the response time of 45 hours produces better forecasts for this particular flood event. Given the suspected loss of water via unobservable routes this result is not surprising. A longer response time would slow the rising of the hydrograph thus making it come closer to the observed one. In any case, it is clear from the above results that the rating curve can be very significant in the real time (even in the short term) prediction of floods since it can alter the shape and peak magnitude of the
Figure III.20. Real time IHFS stage predictions vs corresponding observations for the six sets of API parameters reported in Betson et al. (1969)
Figure III.21. West Otter Creek stage predictions. Sensitivity with respect to the method of rating curve extrapolation to high flow values. A basin response time of 35 hours was used.
Figure III.22. West Otter Creek stage predictions. Sensitivity with respect to the method of rating curve extrapolation to high flow values. A basin response time of 45 hours was used.
predicted hydrograph. Figure C.2.3 presents results corresponding to probability of flooding predictions and is analogous to Figure III.22.

Sensitivity with respect to the response time parameter of IHFS was performed next. The values of 18, 20, 30, 35, and 45 hours were used and Figure III.23 (also Figure C.1.4) together with the previously presented Figures III.21 and III.22 show the IHFS real-time stage predictions and the corresponding observations. Figures C.2.4, C.2.2 and C.2.3 present corresponding results for probability of flooding prediction. The first method of rating curve extrapolation (high flow extrapolation) was used in all cases of Figure III.23. The results point to the larger values of the response time as being more representative of the flood event under study. They also indicate that for this event the IHFS stage predictions are very sensitive to the value of the basin response time, especially during the first part of the event.

Regarding sensitivity to input data, four runs were made involving changes in the data of each of the following IHFS input and output variables: relative humidity, rainfall and stage. For all the cases of this sensitivity analysis the second method of rating curve extrapolation (third-degree polynomial) and a response time of 45 hours were used. A persistent change was introduced in the relative humidity data commencing at 17:00 on 26 April 1988 and ending at 09:00 on 27 April 1988. The change consisted of reducing the relative humidity readings from values near and equal to 100 percent down to 50 percent. This type of sensitivity is based on the formulation of Equation (III.7). The IHFS predictions for this case are presented in Figure III.24 (see also Figure C.1.6). No significant effects on the quality of predictions are discernable. A persistent change was then introduced in the rainfall data. The change introduction can be thought of simulating an error due to a raingage being stuck at a certain reading. Thus, all the readings from 10:00 on 26 April 1988 through the end of the month of April were reduced by 50 percent. The IHFS predictions are presented in Figure III.25 (see also Figure C.1.7). Reduction of the peak stage predictions can be seen. It takes the filter about 32 hours to correct the predictions. It can be seen that a bias in precipitation input that persists over a long period of time can produce a significant bias in the stage predictions of IHFS for basins with a long response time. Finally, isolated changes (or errors) were in turn
Figure III.23. West Otter Creek stage predictions. Sensitivity with respect to the basin response time. Rating curve extrapolation was done using a linear fit to the highest two observed flow values in the rating curve.
Figure III.24. West Otter Creek stage predictions. Sensitivity with respect to persistent changes in relative humidity data. The results correspond to a third-degree polynomial extrapolation of the observed rating curve and a basin response time of 45 hours.
introduced in the stage data. The times of the change introduction were: at 24:00 on 26 April 1988 and at 18:00 on 27 April 1988 and correspond to times near the beginning of the rising limb and near the peak of the stage hydrograph. At each time both -50 percent and +50 percent (of the observed value) errors were introduced. The results are presented in Figures III.26 and III.27 (see also Figures C.1.8 and C.1.9, respectively). It can be seen that negative errors have much more significant effects on the IHFS stage predictions than positive errors. In fact, negative errors in the stage observations are the most influential errors to IHFS stage predictions among all the types of errors considered in this sensitivity analysis. It is imperative therefore that effective quality control exist that guards against these types of errors for cases similar to the West Otter Creek drainage basin and flood examined. Probability of flooding predictions corresponding to the sensitivity analysis results previously presented for stage predictions can be found in Appendix C, Section C.2.

Finally, the magnitudes of the maximum errors in each case is depicted in Table III.5 when updating from both rainfall and stage data is active.
Figure III.25. West Otter Creek stage predictions. Sensitivity with respect to persistent changes in rainfall data. The results correspond to a third-degree polynomial extrapolation of the observed rating curve and a basin response time of 45 hours.
Figure III.26. West Otter Creek stage predictions. Sensitivity with respect to an isolated change in stage data at time 24:00, 26 April 1988. The results correspond to a third-degree polynomial extrapolation of the observed rating curve and a basin response time of 45 hours.
Figure III.27. West Otter Creek stage predictions. Sensitivity with respect to an isolated change in stage data at time 18:00, 27 April 1988. The results correspond to a third-degree polynomial extrapolation of the observed rating curve and a basin response time of 45 hours.
Table III.5

Sensitivity of Stage and Probability of Flooding Predictions to Input Data for the West Otter Creek Flood (Updating from both Rainfall and Stage)

Predictions

<table>
<thead>
<tr>
<th>Case</th>
<th>Stage</th>
<th>Probability of Flooding</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dew-Point</td>
<td>0.00 ft/degree</td>
<td>0.000 degree&lt;sup&gt;-1&lt;/sup&gt;</td>
</tr>
<tr>
<td></td>
<td>(0.0% per degree)</td>
<td></td>
</tr>
<tr>
<td>Rainfall&lt;sup&gt;1&lt;/sup&gt;</td>
<td>0.54 ft/inch</td>
<td>0.07 inch&lt;sup&gt;-1&lt;/sup&gt;</td>
</tr>
<tr>
<td></td>
<td>(16.9% per inch)</td>
<td></td>
</tr>
<tr>
<td>Stage</td>
<td>0.95 ft/foot</td>
<td>0.14 foot&lt;sup&gt;-1&lt;/sup&gt;</td>
</tr>
<tr>
<td></td>
<td>(29.3% per foot)</td>
<td></td>
</tr>
</tbody>
</table>

<sup>1</sup>The results correspond to an average over the period for which the rainfall input error persisted.
IV. CONCLUSIONS AND RECOMMENDATIONS

A. Conclusions

Data from three drainage basins of different areas and hydrologic characteristics were used in an extensive sensitivity analysis. The goal was to ascertain the effects on real time stage predictions of changes in observed data that are processed by an Integrated Hydrometeorological Forecast System (IHFS). The results of this study can be useful in the design of observation networks of sensors of various types for improved capability of operational flood predictions and therefore more effective reservoir operation during flooding conditions. The main conclusions of this study are itemized in the following for each flood event examined.

For the North Fork Big Thompson river flash flood (response time less than 1 hour, and data interval and forecast lead time equal to 1 hour) the following are important conclusions:
1. Isolated dew-point temperature and stage changes had the most pronounced effects on the IHFS stage and probability of flooding predictions.
2. The system component that generates orographic enhancement of rainfall was very important for this flash flood. Significant underestimation of stage results without orographic enhancement of rainfall.
3. Updating from both rainfall and stage results in reducing the time interval over which the effects of isolated input-data changes are "felt" by the IHFS predictions.

For the Tug Fork flood (response time of 13 hours, and data interval and forecast lead time of 3 hours) our conclusions are:
1. Updating from both rainfall and stage data yielded the most accurate and reliable predictions.
2. In case of state updating from both stage and rainfall, isolated errors in the stage data are most detrimental to accurate IHFS stage predictions.

In the case of the West Otter Creek flood (response time greater than 24 hours, with a data interval and forecast lead time of 1 hour) our conclusions are:
1. The rating curve and the response time affect the stage predictions of IHFS significantly.
2. Large isolated errors in stage data have the most pronounced effects on the accuracy of the IHFS stage predictions.
3. When updating from both rainfall and stage is active the time interval over which the effects (that changes in stage data have on stage predictions) are felt by the stage predictions is small.
4. Persistent changes in input rainfall data produce persistent bias in IHFS predictions for basins with a long response time.

Our results summarized in the previous conclusions provide a quantitative basis for the assessment of the utility of data of various types in short-term real-time flood prediction. That is, as the ratio of forecast lead time to basin response time decreases to values less than 0.1, the utility of reliable stage data increases while the utility of the meteorological data and of the rainfall data decreases. Conversely, as the ratio of forecast lead time to basin response time approaches the value of 1, the utility of the meteorological data increases and that of the stage data and of precipitation data decreases. The results suggest that the worth of rainfall as used by IHFS does increase as the ratio decreases. Figure IV.1 confirms these results. There, the results in Tables III.3, III.4 and III.5 are displayed vs. the ratio of forecast lead to basin response time, for the case when updating from both rainfall and stage is active in IHFS. In addition to the above mentioned comments it was clear from our analysis that use of all data types gave the best IHFS performance for all the floods examined and that both the stage and the probability of flood occurrence predictions of IHFS were very good when no changes were introduced in the data.

B. Recommendations

Similar type of sensitivity analysis should be undertaken for other flood events to cover the spectrum of ratios of forecast lead time (and data interval) to basin response time. Cases when soil moisture accounting becomes important (medium intensity storm events with relatively long forecast lead times) should be included in the sensitivity analysis. For those cases it would be necessary to examine the sensitivity of both short- and long-term predictions of IHFS with respect to the particular soil moisture accounting model used. Thus, the API model, the Sacramento model and the Hydrologic Engineering Center HEC-IF model can be used. In addition, in cases when long-
Figure IV.1. Sensitivity of IHFS real-time forecasts with respect to data type and ratio of forecast lead to basin response time. Data from three different basins were used.
term predictions are found sensitive to the soil model, comparison of the predictions of spatially distributed hydrologic models vs. the predictions of spatially lumped models (of the type mentioned above) should be made.

Another issue that deserves special attention in such sensitivity studies is the quantification of the utility of radar data for real-time short- and long-range IHFS forecasts. Various methods of mean areal precipitation estimation from radar data should be used, and comparison of forecasts utilizing such estimates with forecasts that are based only on raingage data should be made. Of particular interest in such studies is the effect of errors in the mean areal precipitation estimates from radar on the reliability of river stage (or discharge) predictions.
V. REFERENCES


APPENDIX A

SENSITIVITY ANALYSIS PLOTS
FOR THE
NORTH FORK BIG THOMPSON RIVER FLASH FLOOD
A.1. Stage Predictions
FIG-A.1.1; BIG THOMPSON RIVER: PH,TD,OR

TIME OF ERROR

STAGE IN FT

10

8

6

4

2

0

18 19 20 21 22 23

TIME IN HOURS; JULY 31, 1976

FIG-A.1.2; BIG THOMPSON RIVER: PH,TD,OR

TIME OF ERROR

STAGE IN FT

12

10

8

6

4

2

0

18 19 20 21 22 23

TIME IN HOURS; JULY 31, 1976

Legend:
- OBSERVATIONS
- NO ERROR
- -2 DEGR
- +2 DEGR
- -5 DEGR
- +5 DEGR
FIG.-A.1.15; BIG THOMPSON RIVER: P,TD,0

STAGE IN FT

TIME OF ERROR

18 19 20 21 22 23

TIME IN HOURS; JULY 31, 1976

FIG.-A.1.16; BIG THOMPSON RIVER: P,TD,0

STAGE IN FT

TIME OF ERROR

18 19 20 21 22 23

TIME IN HOURS; JULY 31, 1976

LEGEND:
- OBSERVATIONS
- NO ERROR
- -2 DEGR
- +2 DEGR
- -5 DEGR
- +5 DEGR
A.2. Probability of Flooding Predictions
FIG.-A.2.5; BIG THOMPSON RIVER: PH,X,OR

TIME OF ERROR

PROBABILITY OF FLOODING

TIME IN HOURS; JULY 31, 1976

FIG.-A.2.6; BIG THOMPSON RIVER: PH,X,0R

TIME OF ERROR

TIME IN HOURS; JULY 31, 1976
APPENDIX B

SENSITIVITY ANALYSIS PLOTS
FOR THE
TUG FORK FLOOD
B.1. Stage Predictions
FIG.-B.1.1; TUG FORK: PH,TD,1 AND 2

TIME ELAPSED IN HRS SINCE 7:00 MAY 6, 1984

TIME OF ERROR

FIG.-B.1.2; TUG FORK: PH,TD,1 AND 2

TIME ELAPSED IN HRS SINCE 7:00 MAY 6, 1984
FIG.-B.1.11; TUG FORK: PH,X,H1

TIME OF ERROR

STAGE IN FT

TIME ELAPSED IN HRS SINCE 7:00 MAY 6, 1984

FIG.-B.1.12; TUG FORK: PH,X,H1

TIME OF ERROR

STAGE IN FT

TIME ELAPSED IN HRS SINCE 7:00 MAY 6, 1984
FIG. B.1.13; TUG FORK: PH,X,H1H2

TIME ELAPSED IN HRS SINCE 7:00 MAY 6, 1984

FIG. B.1.14; TUG FORK: PH,X,H1H2

TIME ELAPSED IN HRS SINCE 7:00 MAY 6, 1984
FIG.-B.1.27; TUG FORK: P,X,H1

STAGE IN FT

TIME OF ERROR

TIME ELAPSED IN HRS SINCE 7:00 MAY 6, 1984

FIG.-B.1.28; TUG FORK: P,X,H1

STAGE IN FT

TIME OF ERROR

TIME ELAPSED IN HRS SINCE 7:00 MAY 6, 1984
B.2. Probability of Flooding Predictions
FIG.: B.2.1; TUG FORK: PH, TD, 1 AND 2

FIG.: B.2.2; TUG FORK: PH, TD, 1 AND 2
APPENDIX C

SENSITIVITY ANALYSIS PLOTS
FOR THE
WEST OTTER CREEK FLOOD
C.1. Stage Predictions
FIG.-C.1.1; SENSITIVITY WITH RESPECT TO API MODEL FOR KANAWA

RESPONSE TIME=45 HOURS

TIME ELAPSED IN HRS SINCE 13:00 APRIL 26,1988
FIG.: C.1.2; SENSITIVITY WITH RESPECT TO RATING CURVE FOR KANAWHA

RESPONSE TIME = 35 HOURS

TIME ELAPSED IN HRS SINCE 13:00 APRIL 26, 1988
FIG.-C.1.3; SENSITIVITY WITH RESPECT TO RATING CURVE FOR KANAWHA

RESPONSE TIME=45 HOURS

STAGE IN FT

OBSERVATIONS
UNIFORM FLOW
3ORD.POLYNOMIAL
HIGH FLOW
LOW FLOW

TIME ELAPSED IN HRS SINCE 13:00 APRIL 26, 1988
FIG.-C.1.4; SENSITIVITY WITH RESPECT TO RESPONSE TIME FOR KANAWHA

HIGH FLOW EXTRAPOLATION OF RATING CURVE

STAGE IN FT

TIME ELAPSED IN HRS SINCE 13:00 APRIL 26, 1988
FIG.-C.1.5; SENSITIVITY WITH RESPECT TO RESPONSE TIME FOR KANAWHA

UNIFORM FLOW COMPUTATIONS FOR RATING CURVE

(TOP WIDTH=100FT, SLOPE=5FT/MILE, n=0.035)

TIME ELAPSED IN HRS SINCE 13:00 APRIL 26, 1988
FIG:C.1.6; PERSISTENT ERROR IN REL.HUMIDITY

(FROM 17:00 ON 26 APRIL TO 09:00 ON 27 APRIL)

INITIAL TIME OF ERROR

STAGE IN FT

TIME ELAPSED IN HRS SINCE 05:00 APRIL 26, 1988

- OBSERVATIONS
- NO ERROR
- PERSISTENCE
FIG.: C.1.7; PERSISTENT ERROR IN PRECIPITATION
(FROM 10:00 ON 26 APRIL TO 24:00 ON 28 APRIL)

INITIAL TIME OF ERROR

STAGE IN FT

0 4 8 12 16 20 24 28 32 36 40 44 48 52 56

TIME ELAPSED IN HRS SINCE 05:00 APRIL 26, 1988

- OBSERVATIONS
- NO ERROR
- PERSISTENCE
FIG:-C.1.8; ERROR IN STAGE READING

TIME ELAPSED IN HRS SINCE 22:00 APRIL 26, 1988
FIG.: C.1.9; ERROR IN STAGE READINGS

TIME OF ERROR

STAGE IN FT

0 1 2 3 4 5 6 7 8 9 10 11 12

TIME ELAPSED IN HRS SINCE 16:00 APRIL 27, 1988

- OBSERVATIONS
- NO ERROR
- -50%
- +50%
C.2. Probability of Flooding Predictions
FIG. C.2.1: SENSITIVITY WITH RESPECT TO API MODEL FOR KANAWA

RESPONSE TIME - 45 HOURS

TIME ELAPSED IN HRS SINCE 13:00 APRIL 26, 1988

PROBABILITY OF FLOODED
FIG.-C.2.2; SENSITIVITY WITH RESPECT TO RATING CURVE FOR KANAWHA

RESPONSE TIME=35 HOURS

TIMESTAMP: 13:00 APRIL 26, 1988

PROBABILITY OF FLOODING

0.0 0.2 0.4 0.6 0.8

0 4 8 12 16 20 24 28 32 36 40 44 48 52

- UNIFORM FLOW
- 3ORD.POLYNOMIAL
- HIGH FLOW
- LOW FLOW
FIG.: C.2.3; SENSITIVITY WITH RESPECT TO RATING CURVE FOR KANAWHA

RESPONSE TIME=45 HOURS

PROBABILITY OF FLOODING

TIME ELAPSED IN HRS SINCE 13:00 APRIL 26, 1988
FIG: C.2.4; SENSITIVITY WITH RESPECT TO RESPONSE TIME FOR KANAWHA

HIGH FLOW EXTRAPOLATION OF RATING CURVE

TIME ELAPSED IN HRS SINCE 13:00 APRIL 26, 1988
FIG.: C.2.5; SENSITIVITY WITH RESPECT TO RESPONSE TIME FOR KANAWHA

MANNING'S FORMULA USED

(TOP WIDTH=100FT, SLOPE=5FT/MILE, n=0.035)

TIME ELAPSED IN HRS SINCE 13:00 APRIL 26, 1988
FIGURE C.2.7: PERSISTENT ERROR IN PRECIPITATION
(FROM 10:00 ON 26 APRIL TO 24:00 ON 28 APRIL)

INITIAL TIME OF ERROR

PROBABILITY OF FLOODING

TIME ELAPSED IN HRS SINCE 05:00 APRIL 26, 1988

- NO ERROR
- PERSISTENCE
FIG: C.2.8: ERROR IN STAGE READINGS

TIME ELAPSED IN HRS SINCE 22:00 APRIL 26, 1988

Probability of Flooding

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