PHASE-AVERAGED NOMINAL WAKE FOR SURFACE SHIP
IN REGULAR HEAD WAVES

by

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ABSTRACT

Phase-averaged organized oscillation velocities \( (U,V,W) \) and random fluctuation Reynolds stresses \( (\overline{uu}, \overline{vv}, \overline{ww}, \overline{uv}, \overline{uw}) \) are presented for the nominal wake of a surface ship advancing in regular head (incident) waves, but restrained from body motions, i.e., the forward-speed diffraction problem. A 3 × 3 × 100 m towing tank, plunger wave maker, and towed, 2D particle-image velocimetry (PIV) and servo mechanism wave-probe measurement systems are used. The geometry is DTMB model 5415 \( (L = 3.048 \text{ m}, 1/46.6 \text{ scale}) \), which is an international benchmark for ship hydrodynamics. The conditions are Froude number \( Fr = 0.28 \), wave steepness \( Ak = 0.025 \), wavelength \( \lambda / L = 1.5 \), wave frequency \( f = 0.584 \text{ Hz} \), and encounter frequency \( f_e = 0.922 \text{ Hz} \). Innovative data acquisition, reduction, and uncertainty analysis procedures are developed for the phase-averaged PIV. The unsteady nominal wake is explained by interactions between the hull boundary layer and axial vortices and incident wave. There are three primary wave-induced effects: pressure gradients \( 4\% U_c \), orbital velocity transport \( 15\% U_c \), and unsteady sonar dome lifting wake. In the outer region, the uniform flow, incident wave velocities are recovered within the experimental uncertainties. In the inner, viscous-flow region, the boundary layer undergoes significant time-varying upward contraction and downward expansion in phase with the incident wave crests and troughs, respectively. The 0\textsuperscript{th} harmonic exceeds the steady-flow amplitudes by 5-20\% and 70\% for the velocities and Reynolds stresses, respectively. The 1\textsuperscript{st}-harmonic amplitudes are large and in phase with the incident wave in the bulge region (axial velocity), damped by the hull and boundary layer and mostly in phase with the incident wave (vertical velocity), and small except near the free surface-hull shoulder (transverse velocity). Reynolds stress amplitudes are an order-of-magnitude smaller than for the velocity components showing large values in the thin boundary layer and bulge regions and mostly in phase with the incident wave.
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1. INTRODUCTION

Procurement of detailed global and local flow benchmark experimental fluid dynamics (EFD) data for fluid physics, model development, and validation of Reynolds-averaged Navier-Stokes (RANS) ship hydrodynamics computational fluid dynamics (CFD) codes has been an ongoing effort since ca. 1970 with data used periodically at workshops wherein codes are compared with each other and the data. Recent efforts have focused on modern tanker (KRISO VLCC, VLCC2), container (KRISO KCS), and surface combatant (DTMB 5415) hull forms and more challenging test cases, as per the Gothenburg 2000 Workshop (Larsson et al., 2003) and Tokyo 2005 Workshop (Hino, 2005), the latter of which the present data is used. Kim et al. (2001) and Lee et al. (2003) provide steady-flow data for VLCC2 and KCS. The present interest is in DTMB 5415, for which data procurement has been part of an international collaboration between IIHR1, INSEAN2, and DTMB3 over the past ten years. Initially steady-flow data is procured, including rigorous uncertainty analysis (Longo and Stern, 2005); identification of facility biases (Stern et al., 2000 and 2005); mean flow map (Olivieri et al., 2001); steady nominal wake PIV (Gui et al., 2001b); and propeller-hull interaction (Ratcliffe, 2001). Subsequently, unsteady-flow data is procured, including wave breaking (Olivieri et al., 2004) and forward-speed diffraction forces, moment, and wave pattern (Gui et al., 2001c and 2002). The present paper concerns the forward-speed diffraction problem for unsteady PIV at the nominal wake of 5415. The most recent efforts have focused on free roll decay measurements with and without bilge keels (Felli et al., 2004; Bishop et al., 2004; Irvine et al., 2004).

In parallel, PIV studies for ship velocity fields have been conducted for various specialized purposes, as reviewed by Longo et al. (2004a). Fu et al. (2002) apply digital PIV and the auto-correlation evaluation method in a rotating arm basin to study dominant cross-flow separation induced by a 5.18 m submarine model in a turn. Dong et al. (1997) apply film-based PIV with the auto-correlation method to investigate the bow flow of a 3.05 m ship model in a towing tank. PIV images at several axial stations in the bow wave highlight the cross plane vector fields and considerable vorticity entrained into the toe of

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the bow wave. Roth et al. (1999) apply digital PIV and the cross-correlation method to study the mean and turbulent bow flow of a 7.01 m ship model at several axial stations on the fore body including convergence tests on mean and turbulence variables and effects of interrogation window size on turbulence structure and statistics. Di Felice and De Gregorio (2000) use digital PIV and the cross-correlation method to investigate the turbulent wake of a 5.41 m ship model equipped with two, 4-bladed propellers in a circulating water channel. By synchronizing the PIV recordings with the propeller shaft, phase-averaged velocities and vorticity are computed from several image pairs at a range of phase angles. Calcagno et al. (2002) use 3D, stereoscopic PIV in a circulating water tunnel to investigate the turbulent propeller wake flow of a 6.096 m ship model equipped with a 0.222 m diameter, 5-bladed propeller. The phase-averaged data highlights the interactions of the turbulent wake of the hull and propulsor, tip vortex system, slipstream contraction, and strong diffusion and dissipation of the propeller blade wakes. Cotroni et al. (2000) and Di Felice et al. (2000) investigate the phase-averaged wake flow of two, 4-bladed propellers in a cavitation tunnel using digital PIV, the cross-correlation evaluation method, and uncertainty assessment. Phase-averaged data are reported in sufficient detail to track the tip vortex systems including formation and breakdown with increasing downstream distance from the propeller disk. Judge et al. (2001) investigate tip leakage vortices from a 0.8504 m diameter, 3-bladed, ducted rotor with digital PIV and the cross-correlation evaluation method in a variable-pressure water tunnel. Similar studies that are more recent include PIV analysis of flow around a container ship model with a rotating propeller by Paik et al. (2004) and three-component velocity field measurements of propeller wake using stereo PIV by Lee et al. (2004).

Also of interest are unsteady PIV studies for more fundamental geometries. Lam and Leung (2005) use phase-averaged PIV to study the asymmetric turbulent vortex shedding of a flat plate at high incidence. The vortex street is comprised of a train of leading edge vortices alternating with a train of trailing edge vortices. The trailing edge vortices possess higher peak vorticity and Reynolds stress production. The results at three angles of attack collapse into similar trends by using the projected plate width as the characteristic length scale. Konstantinidis et al. (2005) use a conditional averaging approach to study the natural and forced turbulent wake of a circular cylinder. Inflow
oscillations are shown to affect the vortex formation and shedding. Wernert and Favier (1999) investigate principles of the phase-averaging technique for PIV with regard to convergence criterion with application to pitching airfoils oscillating through dynamic stall. In a series of papers (most recently, Uzol et al., 2003), unobstructed PIV within an axial turbo-pump using liquid and blades with matched refractive indices is used to study phase-averaged velocities and turbulence and average passage flow field and deterministic stresses. Wake-blade and wake-wake interactions induce flow non-uniformities, turbulent hot spots, and high deterministic stresses. Highest levels of phase-dependent unsteadiness and deterministic stresses are in the tip region induced by the tip vortex.

The motivation of the present study is to provide benchmark EFD data for advancement of ship hydrodynamics unsteady RANS codes from steady to unsteady flow. The forward-speed diffraction problem, i.e., ship advancing in regular head (incident) waves but restrained from motions is identified as a building block problem; since, in traditional potential flow strip theory approaches, the exciting forces for motions are the solution to the forward-speed diffraction problem. The ability of unsteady RANS for the forward-speed diffraction problem is viewed as a first step in merging the traditionally separate fields of resistance and propulsion with seakeeping and maneuvering and ultimately realization of simulation based design.

The approach is complementary CFD, EFD, and uncertainty assessment. CFD is used to guide EFD, EFD is used for validation and model development, and lastly CFD is validated and fills in sparse data for complete documentation and diagnostics of the flow. The EFD includes towing tank tests using DTMB model 5512 (length $L = 3.048$ m geosym of DTMB model 5415) and nominal wake phase-averaged PIV and uncertainty analysis. The model geometry is shown in Fig. 1 and its parameters are given in Table 1. Test conditions are based on previous steady-flow PIV (Gui et al., 2001b) and unsteady-flow force, moment and wave pattern (Gui et al., 2001c and 2002) and phase-averaged regular head wave PIV (Longo et al., 2004b) studies, i.e., Froude number $Fr = 0.28$, wave steepness $Ak = 0.025$, wave length $\lambda / L = 1.5$, wave frequency $f = 0.584$ Hz, and encounter frequency $f_e = 0.922$ Hz. The test conditions (medium speed, low wave steepness, and long wavelength) are selected to produce primarily a first harmonic linear
response in the forces and moment and wave and flow fields. Previous (Rhee and Stern 2001) and concurrent (Carrica et al., 2005) CFD studies are used for estimating the measurement regions and analysis of the data. Innovative data acquisition, reduction, and uncertainty analysis procedures are developed for the phase-averaged PIV, as part of the project. Appendices A and B provide supplemental figures and Longo et al. (2004b), respectively.

2. TEST DESIGN

2.1. Facility and coordinate system

The tests are conducted in the IIHR towing tank, as shown in Fig. 2a. The tank is 100 m long, 3.048 m wide and 3.048 m deep, and equipped with a drive carriage and model trailer, plunger-type wave maker, automated wave dampener system, and wave-dampening beach. The drive carriage is instrumented with two data acquisition computers, speed circuit, and signal conditioning for PIV, wave elevation, and carriage speed measurements. The drive carriage pushes a 5.5 m trailer which is used as a platform for the PIV system and point of attachment for models. The wave maker is hydraulically driven and controlled with an MTS controller and LabView software. It is capable of producing a variety of regular waves with a range of $\lambda = 0.5-6.0$ m and $Ak = 0.025-0.3$ and can also generate irregular waves. The wave dampener system consists of a double-row of 10.2 cm diameter swimming pool lane markers on both sides of the tank, pulleys, and electric winch. The wave dampeners are raised prior to unsteady measurement carriage runs which allow waves to propagate unobstructed through the tank. After the carriage run, the wave dampeners are lowered to dampen free-surface disturbances. For steady measurement carriage runs, the wave dampeners remain lowered before and after carriage runs. The wave dampeners enable twenty- and twelve-minute intervals between unsteady and steady measurement carriage runs, respectively. These time intervals are determined sufficient based on visual inspection of the free surface and also on PIV data taken after steady and unsteady carriage runs such that the measured residual motions are less than the PIV uncertainties and close to the noise level of the measurement system. The noise level of the PIV system is previously determined
from static (carriage speed $U_c = 0 \text{ m/s}$) and uniform flow ($U_c = 1.53 \text{ m/s}$) tests, as given in Table 2 and discussed later.

A right-handed Cartesian coordinate system fixed to the model is used for the tests with the origin at the intersection of the calm free surface and forward perpendicular of the model. The $x$, $y$, $z$ axes are directed downstream, transversely to starboard, and upward, respectively, as shown in Fig. 2b. The coordinates, PIV velocities, and other variables of interest are non-dimensional using $L$ and $U_c$.

2.2. Model

The model geometry is DTMB model 5512, a 1:46.6 scale, $L = 3.048 \text{ m}$, fiber-reinforced Plexiglas hull with block coefficient, $C_B = 0.506$. DTMB model 5512 is a geosym of DTMB model 5415, which is a 1:24.8 scale, $L = 5.72 \text{ m}$ model ship conceived by the USA Navy as a preliminary design for a surface combatant ca. 1980 with a sonar dome bow and transom stern. The model is un-appended for the current tests, i.e., not equipped with shafts, struts, bilge keels, propulsors, or rudders. To initiate transition to turbulent flow, a row of cylindrical studs of 1.6 mm height and 3.2 mm diameter are fixed with 9.5 mm spacing at $x = 0.05$. The stud dimensions and placement on the model are in accordance with the recommendations by the 23rd ITTC (ITTC, 2002). PIV measurements are made on the port side of the model, which is painted black for minimization of laser-sheet reflection.

2.3. Conditions

Unsteady (with wave) and steady (without wave) tests are performed at $U_c = 1.53 \text{ m/s}$. The model is rigidly fixed to the carriage with zero yaw and roll angle and towed at the dynamic sunk and trimmed condition corresponding to $Fr = 0.28$ (Longo and Stern 2005): $\Delta FP = -0.00310L$, $\Delta AP = -0.000734L$. For unsteady tests, $\lambda = 4.572 \text{ m}$, $f = 0.584 \text{ Hz}$, and $Ak = 0.025$ where $f$ and $Ak$ are defined in equations (1) and (2), and $A$ and $g$ are wave amplitude and local gravity acceleration ($g = 9.8031 \text{ m/s}^2$), respectively.

\[
f = \sqrt{\frac{g}{2\pi\lambda}}
\]  
(1)
The encounter frequency is given by

$$f_e = \sqrt{\frac{g}{2\pi\lambda}} + \frac{U_c}{\lambda} = 0.922 \text{ Hz}$$

which is the dominant frequency of the unsteady response. The speed and wave conditions are based on Gui et al. (2001b, 2001c, 2002). The Froude number $Fr = \frac{U_c}{\sqrt{gL}}$ is 0.28, which corresponds to the cruise speed for the full-scale ship. The regular head wave parameters are selected following observation and analysis of unsteady forces and moment results because these parameters produce the most manageable linear response in the far field of the ship model.

### 2.4. Data acquisition and reduction methodology

The present interest is in PIV measurements of phase-averaged organized oscillation velocities ($U, V, W$) and random fluctuation Reynolds stresses ($\overline{uu}, \overline{vv}, \overline{ww}, \overline{uv}, \overline{uw}$); hereafter, referred to as phase-averaged (or simply) velocities and Reynolds stresses. The regular head wave primarily produces a first harmonic response; therefore, following Gui et al. (2001c, 2002), a Fourier series (FS) reconstruction is desired. The FS reconstruction is based on a 5th-order least-squares regression (LSR) method used to interpolate the instantaneous measurements acquired at random phases for a continuous curve for all phases. There are three reasons for using this procedure: (1) It is difficult to synchronize the trigging of the PIV data acquisition with the phase of the regular head wave; (2) The uncertainty of the regular head wave frequency is large compared to the phase interval of data acquisition; and (3) The PIV data acquisition rate is not an integer fraction of the encounter frequency. Fig. A1 provides a block diagram showing the data acquisition and reduction methodology and procedures. The 5th-order LSR is determined as a best fit to the data based on trial and error analysis using up to a 10th-order LSR. After the experiments are completed and analyzed some limited testing is done using an
arithmetic mean moving average method to test the accuracy of the LSR, as discussed later.

2.4.1. Instantaneous measurements. The data-reduction equation for an instantaneous PIV measurement is given in equation (4). \( L_{\text{obj}} \) is the width or height of the object plane (measurement area), \( L_{\text{img}} \) is the width or height of the CCD array of the digital camera, \( \Delta t \) is the time between successive PIV frames, \( U_c \) is the carriage speed, and \( S_{k,i} \) is the instantaneous particle displacement in a given interrogation area determined by the correlation software.

\[
C_{k,i} = \frac{L_{\text{obj}}}{L_{\text{img}} \Delta t U_c} S_{k,i} \tag{4}
\]

The axial \( U \), transverse \( V \), and vertical \( W \) velocities are given by \( k = 1, 2, 3 \), respectively. In equation (4) and (5)-(7) that follow, the measurement sample number is given by \( i = 1, \ldots, N_f \) where \( N_f \) is the number of valid measurements at a given grid point in a carriage run or group of carriage runs after filtering from a population of \( N_r \) PIV recordings. The instantaneous normal and shear Reynolds stresses are computed with equations (5) and (6), respectively, where \( i \) and \( N_f \) are applicable same as for equation (4).

\[
(c_k c_k)_i = (C_{k,i} - C_k)^2 \tag{5}
\]

\[
(c_m c_n)_i = (C_{m,i} - C_k)(C_{n,i} - C_k) \tag{6}
\]

The axial \( \overline{uu} \), transverse \( \overline{vv} \), and vertical \( \overline{ww} \) normal Reynolds stresses are given by \( k = 1, 2, 3 \), respectively, in equation (5). The \( \overline{uv} \) and \( \overline{uw} \) shear Reynolds stresses are given by \( m \) and \( n \) subscripts \( m = 1, 2 \) and \( m = 1, n = 3 \), respectively, in equation (6). \( \overline{vw} \) is not obtainable with the 2D PIV system. For a steady-flow application, i.e., ship advancing into calm water, \( C_k \) in equations (5) and (6) is the average velocity component determined with the average particle displacement component \( S_k \) as

\[
C_k = \frac{L_{\text{obj}}}{L_{\text{img}} \Delta t U_c} \frac{1}{N_f} \sum_{i=1}^{N_f} S_{k,i} = \frac{L_{\text{obj}}}{L_{\text{img}} \Delta t U_c} S_k \tag{7}
\]
For an unsteady-flow application, i.e., ship advancing into regular head waves, $C_k$ in equations (5) and (6) is a LSR polynomial $X_{LSR}$, where $X_{LSR}$ is representative of the response of the unsteady velocity through one encounter period. This will be described in more detail below.

The data-reduction equation for the regular head wave elevation measurement is

$$ζ_1(t_i) = \frac{z_1(t_i)}{L}$$

where $z_1(t_i)$ is the $i^{th}$ regular head wave elevation (dimensional) in a series of $N_r$ sweeps of an AD card. This measurement occurs synchronously with the acquisition of $N_r$ PIV recordings. Equation (9) is the phase expression for the unsteady PIV measurements at the nominal wake plane of model 5512.

$$γ_i = γζ_1 + 2π \frac{D}{λ} - 2πt_i f_e$$

The components of equation (9) are shown in Fig. 3. The first term on the right hand side is the 1st-harmonic phase of the incident wave which is derived from $ζ(t_0)$ at $t = 0$ sec (when data acquisition commences in the carriage run) at the position of a servo wave gage upstream of the PIV system. The second term on the right hand side is the phase delay created by placing the servo wave gage upstream of the measurement area by a distance $λ/D$. The third term on the right hand side is the phase delay associated with the regular time interval between successive PIV recordings. For this application, $t_i$ is incremented by 133 ms since the PIV system acquisition rate of vector fields is 7.5 Hz. This phase delay is expressed as a fraction of the encounter period $T = 1/f_e$.

2.4.2. Least-squares regression. The functional relationships between the instantaneous phase-averaged velocities or Reynolds stresses and their corresponding phases are modeled with a LSR polynomial curve fit. A numerical implementation of the method of least squares is used to compute the polynomial curve fits. The mathematical expression for an $n^{th}$-order polynomial curve fit is given by
where \( X_{LSR,j} \) and \( \gamma_j \) for \( j = 1, \ldots, 360 \) are the regression ordinate and abscissa, respectively, and the \( a' \)'s are regression coefficients which are functions of the variables in equations (4)-(6) and (9). \( \gamma \) is used in the analysis to reconstruct the LSR model for whole phase angles in the range \( \gamma_j = 1^\circ\text{-}360^\circ \).

### 2.4.3. Fourier series expansion.

The LSR polynomial curve fits for the phase-averaged velocities and Reynolds stresses are reconstructed with a FS expansion. A numerical implementation of the FS analysis is used. The mathematical expression for an \( N^{th}\)-order FS model representing a phase-averaged velocity or Reynolds stress polynomial curve fit through one encounter period is

\[
X_{FS,j} = \sum_{n=0}^{N} A_n \cos(2\pi f t_j + \gamma_n) \tag{11}
\]

where \( A_n \) and \( \gamma_n \) are \( n^{th}\)-order harmonic amplitude and phase, respectively, \( t_j \) is the time in the encounter period for \( j = 1, \ldots, 360 \) when the FS is computed, and \( \gamma_0 = 0^\circ \). Summations are used to compute the FS coefficients in equations (12)-(13), taking advantage of the fact that the LSR modeling is for 360 equidistant points over the encounter period \( T \). The amplitude \( A_n \) and phase \( \gamma_n \) of the FS are expressed as:

\[
A_n = \frac{2}{360} \left[ \left( \sum_{j=1}^{360} X_{LSR,j} \cos(2\pi f t_j) \right)^2 + \left( \sum_{j=1}^{360} X_{LSR,j} \sin(2\pi f t_j) \right)^2 \right]^{1/2} \tag{12}
\]

\[
\gamma_n = \tan^{-1}\left[ \frac{\sum_{j=1}^{360} X_{LSR,j} \sin(2\pi f t_j)}{\sum_{j=1}^{360} X_{LSR,j} \cos(2\pi f t_j)} \right] \tag{13}
\]

where the \( A_n \)'s and \( \gamma_n \)'s are the final, desired results for the present study.
2.5. Measurement systems

The Dantec towed, 2D PIV system is shown in Figs. 2, 3 and 4. The PIV hardware components (hydrodynamic strut, laser, light-guiding arm, light-sheet optics, digital camera) are assembled on a massive 2D, computer-controlled traversing system capable of automated movement along the transverse $y$ and vertical $z$ axes. Movements in the $x$-coordinate are manual. The strut is pressurized, partly submerged, and contains a 20 mJ, dual cavity Nd:Yag laser and light-guide arm for steering 532 nm beams through the light-sheet optics, which is housed in a submerged, streamlined torpedo. The digital camera is a one-megapixel $1008 \times 1018$ pixels cross-correlation camera fitted with a f/1.4 50 mm lens that views the light sheet from a distance of 50 cm through a 90° mirror. The maximum object-plane size with the 50 mm lens is $7.5 \times 7.5$ cm$^2$, however, smaller areas can be utilized to realize a number of advantages which will be discussed later. The camera is housed in a separate submerged, streamlined torpedo. Light-sheet and camera torpedoes are joined with a rigid, streamlined mini-strut such that the light sheet is orthogonal to the viewing axis of the camera. Fig. 4 shows the system configured to measure in two modes. In the first, the mini-strut is horizontal and velocities ($U$, $W$) and Reynolds stresses ($\overline{uu}, \overline{ww}, \overline{uw}$) are acquired in vertical $xz$ planes. In the second, the mini-strut is rotated downward through 90° whereby velocities ($U$, $V$) and Reynolds stresses ($\overline{uu}, \overline{vv}, \overline{uv}$) are acquired in horizontal $xy$ planes. Synchronization of the laser and camera, image processing, and acquisition of towing carriage speed are performed with the Dantec PIV 2000 vector processor which is equipped with a four-channel, 12-bit analog-to-digital (AD) card. Data acquisition and parameter settings are facilitated with an IBM-compatible, Windows NT PC equipped with a National Instruments GPIB card and DANTEC v.3.11 Flowmanager software. Results in the form of vector maps are displayed real-time at a rate of 7.5 Hz. Unsteady data is phase-locked to the regular head wave elevation by connection of a servo wave probe to the PIV AD board. The probe monitors the regular head wave from a distance $D = 4.42$ m upstream of the measurement area. The servo probe is a ±5 cm, pre-calibrated Kenek wave probe with a resolution of 0.1 mm and 1:1 frequency response up to 5 Hz for the present incident wave amplitudes. Silver-coated hollow glass spheres with a density of 1600 kg/m$^3$ and an average diameter
of 15 µm are used as seed particles. These particles have demonstrated very good light-reflectance for PIV image capture and adequate suspension capability. Additionally, the particles are capable of following sinusoidal motions with frequencies up to 1375 Hz.

The second measurement system is used for monitoring and measuring the carriage speed and servo wave probe output for each data-acquisition run. It is composed of a DOS PC, IIHR-fabricated speed circuit, and Kenek servo wave gage. The speed circuit hardware includes an 8000-count optical encoder affixed to a wheel of the drive carriage through a pair of chain-driven sprockets and a digital-to-analog converter. A detailed uncertainty analysis of the IIHR speed circuit has estimated the uncertainty in $U_c$ of $U_{uc} = 0.25\%$ for towing speeds corresponding to $Fr = 0.28$ with model 5512 (Longo and Stern, 2005).

2.6. Data acquisition and reduction procedures

2.6.1. Data acquisition setup. The measurement area dimensions are 192 × 1018 pixels (14.3 × 74.9 mm) or 18% of the total field of view. Advantages of this measurement area include previous use by Gui et al. (2001b), real-time data throughput for accurate vector map time stamping, and reduction of amplitude and phase errors at the periphery of the image plane (Longo et al., 2002a). Other PIV data acquisition parameters include 32 × 32 pixel interrogation areas, 50% overlap in both coordinates, and window-offsetting 8 pixels in the axial coordinate. A Gaussian window function is used in the correlations. With the above settings, the measurement grid is 11 × 62 vectors. PIV image pairs or recordings are taken at 133 ms intervals or 7.5 Hz. The time between successive PIV images is $\Delta t = 490\mu$s. Measurements are taken at several measurement area locations within six zones. For measurements in the vertical plane, 21 measurement area locations are used in zones A, B, C (Fig. 4, inset #1). For measurements in the horizontal plane, 16, 17, and 21 measurement area locations are used in zones D, E, and F (Fig. 4, inset #2), respectively. All zones are centered axially on the nominal wake plane and cover the region of interest in the $yz$ cross plane predicted by a RANS solution (Carrica et al., 2005) for the current test conditions, i.e., $x = 0.935; -0.06 \leq y \leq 0; -0.06 \leq z \leq 0$. Zones are arranged to provide adequate overlap for measurement-continuity checks across zone boundaries, i.e., 28-80% between zones A
and B, 28% between zones B and C, and 32% between zones D, E, F. Zone A sets the nearest measurement locations to the model through placement of the top interrogation area at \( x = 0.935 \) adjacent to the hull surface. This ensures a minimum distance of 1.2 mm between the center of the nearest interrogation area and the hull surface or \( y^+ = 70 \) based on a friction velocity estimate from flat plate data. For the unsteady cases, regular head wave data is taken \( D = 4.42 \) m upstream of the measurement area midpoint. \( U_c \) and \( \zeta(t) \) are sampled on the DOS PC for 10 seconds at a rate of 410 Hz.

### 2.6.2. Data acquisition procedures.

For unsteady data acquisition, first at-rest reference voltages for \( U_c \) and \( \zeta(t) \) are measured. Then, the sidewall dampeners are raised, the wave maker is started, and enough time is allowed to elapse for a fully-developed series of waves to travel across the length of the tank. When this condition occurs, the carriage is started and accelerates through 10 m to a constant speed. Data acquisition commences after traveling another 10 m which allows the unsteady free surface and flow field to develop and reach a state where they are not in transition. Data acquisition occurs for 27 seconds or 42 m through the tank after which the carriage is decelerated. A single carriage run produces 200 PIV recordings. Each PIV recording is accompanied by a simultaneous sweep across the analog inputs of the PIV processor such that the wave elevation from the servo wave probe can be logged. The wave elevation at \( t = 0 \) sec on the first PIV recording determines the initial phase of the regular head wave while wave phases for subsequent PIV recordings are determined with equation (9). After a PIV image pair and the analog inputs are sampled, the PIV processor completes the image correlation, computes a vector field, and then repeats the process \( N_r - 1 \) times in real time as the carriage run progresses. The DOS computer runs in parallel with the PIV Windows computer, acquiring \( U_c \) and \( \zeta(t) \) data through the initial one-third of the carriage run. \( \zeta(t) \) from the DOS computer is an independent check for the PIV-acquired wave elevation measurements. Vector maps accumulate at the rate of 200 maps per carriage run over ten and six carriage runs for unsteady and steady cases, respectively.

Prior to initiating the test program, a convergence study sets the required number of carriage runs that yield converged \( 0^{th} \)- and \( 1^{st} \)-harmonic amplitude and phases. The study focuses on convergence histories and running means for a single measurement area in the
viscous flow (zone B; plane 05). Fig. 5 shows typical convergence results for phase-averaged axial velocity at a grid point in the viscous flow region marked with a red dot in Fig. 4. Fig. 5 shows $U_0$, $U_1$, $\gamma U_1 N^{th}$ value, running mean, and convergence parameter $X_{cnv}$ histories with increasing number of valid data samples. The convergence parameter is defined by

$$X_{cnv} = \left( \frac{X_n - X_{n-1}}{DX} \right) \times 100\%$$  \hspace{1cm} (14)

where $X_n$ for $n = 2, \ldots, N_f$ is any phase-averaged velocity or Reynolds stress. Equation (14) evaluates successive changes in the value of $X$ at a given grid point as a percentage of the dynamic range $DX$ for increasing numbers of valid PIV samples at that grid point. Convergence is considered to be achieved for values of $X_{cnv}$ that are within the uncertainty level $U_X$ of $X$. At the test program conclusion, a more extensive convergence study is undertaken to evaluate the convergence parameter (14) of all phase-averaged velocities and Reynolds stresses at all measurement areas and grid points, as shown in Fig. A2-A3. Results indicate average convergence parameter values of 1.0%, 4.1% and 4.5% for 0th- and 1st-harmonic amplitudes and 1st-harmonic phase, respectively, as shown in Table 2.

**2.6.3. Data reduction procedures.** Unsteady and steady data is post-processed with FORTRAN programs that are written and run on a Windows PC. Unsteady data is phase averaged by processing batches of PIV and corresponding DOS data files acquired at single light sheet positions. From a batch, $f_e$ is computed from PIV-sampled $\zeta_I$ for each carriage run followed by FS analysis which yields $\zeta_{II}$ and $\gamma_{II}$ at $t = 0$ sec. $f_e$, $\gamma_{II}$, and $D$ are then used in equation (9) to compute the specific phase angle of all vector maps in each carriage run (Fig. 6a). The following procedures are used at all grid points in the measurement area. Data is sorted on phase angle from 0-2$\pi$ (Fig. 6b) and then filtered with a two-stage range filter and 2D-median filter to remove spurious vectors (Fig. 6b, c). Because there is no penalty in the phase-averaging process, rejected vectors are not replaced. A 5th-order LSR curve is fit to the filtered data, which represents the phase-averaged unsteady response through one encounter period. Then, a 10th-order FS is
computed from the LSR curve fit to obtain the harmonic amplitudes and phases of the phase-averaged response (Fig. 6d). Phase-averaged Reynolds stresses are computed in the range 0-2π as differences between the instantaneous velocities and the FS reconstruction of the velocities. Similarly as with the organized oscillations, a 5th-order LSR is fit to the random fluctuations followed by a 10th-order FS. Convergence histories for 0th- and 1st-harmonic amplitude and 1st-harmonic phase are computed and monitored at seven locations longitudinally across the measurement area. When all raw data is processed as per the above procedures, constant-y or -z data is patched across zone boundaries and five passes of a moving-average filter is applied across the range of y or z values to remove high-frequency noise in the patch regions (Fig. A4). The three-dimensional flow field at the nominal wake plane is constructed through linear interpolation of xz and xy data to a standard 150 × 150 grid in the region -0.06 ≤ y ≤ 0; -0.06 ≤ z ≤ 0. Animations of the velocities and Reynolds stresses are generated with equation (11).

Steady data is also reduced by processing batches of PIV and corresponding DOS data files acquired at single light sheet positions. From a batch, spurious vectors are removed with a two-stage range filter and a 3D median filter where the third dimension is time. The velocities and Reynolds stress results are computed with statistical analysis at each grid point from the population of valid vectors. Data patching between zones and convergence of the results is computed and monitored as per the unsteady case. Evaluation of the new data-reduction software for steady PIV is done by comparing reprocessed original data with published results from Gui et al. (2001b) and current steady-flow data at a single xy cut from zones D, E, F at -0.06 ≤ y ≤ 0, z = -0.025 (Fig. A5). Differences in reprocessed original data with published results are likely caused by small differences in the range and 3D-median data filtering. Differences in the current steady-flow data and published results are generally small but significant for $\bar{u}u$ and $\bar{v}v$ in high-gradient regions and likely caused by differences in model roughness, laser intensity, seeding density, camera focus, or $N_f$ between original and current experiments. The closeness of overlap between zones D/E and E/F is generally good for $U$, $V$, $\bar{u}v$ but degraded somewhat for $\bar{u}u$ and $\bar{v}v$ in the outer region where transition between high- and low-turbulence areas occurs. This effect may be caused by PIV-image distortion or
interaction between the submerged part of the PIV system and the free surface and/or hull surface.

At the test program conclusion, an analysis is made of the data reduction LSR interpolation procedure by comparison with interpolation using an arithmetic mean based on a moving average method where averages between $3^\circ$-$358^\circ$ are based on local averages between $\pm 2^\circ$. Table A1 shows comparison between the LSR and arithmetic mean methods for the outer flow region with and without the model. The differences for the $1^\text{st}$-harmonic amplitude and phase are within the measurement uncertainties. The differences for the $2^\text{nd}$-harmonic amplitude are fairly large but although not estimated the uncertainty is also expected to be large such that re-analysis of all the data is not deemed necessary.

3. UNCERTAINTY ANALYSIS

Uncertainty analysis of the steady and unsteady nominal wake measurement results follows ASME (1998) and AIAA (1999). The procedures are based on estimates of systematic bias and random precision limits, and their root-sum-square (RSS) combination to ascertain total uncertainty. Ninety-five percent confidence levels are achieved through careful estimation of bias errors and usage of a small sample, multiple test approach for precision errors.

Original development of uncertainty analysis procedures for steady PIV measurements is undertaken as part of commissioning procedures for the IIHR towed, 2D PIV system and documentation of the quality of nominal wake data (Gui et al. 2001b). Techniques for reduction of PIV cross-correlation evaluation bias with window functions are also developed (Gui et al. 2001a). Subsequently, uncertainty analysis procedures are developed for unsteady forces and moment and wave field data (Gui et al. 2001c; Gui et al. 2002) and extended for present unsteady PIV measurements by Longo et al. (2002b). More recently, improvements are made to directly account for the bias errors in the LSR representation of the unsteady PIV data.
3.1. Steady uncertainty analysis

Measurement uncertainties for the velocities and Reynolds stresses are provided in Table 2 including original values from Gui et al. (2001b). Bias and precision limit contributions to the total uncertainties are provided only for the current results. The current results are considered satisfactory and show 1-3% reduced uncertainties over previous values for six of eight variables with $\overline{w\dot{w}}$ and $\overline{u\dot{v}}$ 0.3% and 1.8% higher, respectively, than previously. Reductions are a result of improved repeatability of the measurements, which lowers the precision limits and thereby $U_X$. For the velocities, more than half of the uncertainty is attributed to the bias limits, whereas, the precision limits are dominant in the Reynolds stress uncertainties. This result is expected since the signal-to-noise ratio of the Reynolds stresses is lower in comparison to that of the velocities, thus making the repeatability of the Reynolds stress result more difficult.

3.2. Unsteady uncertainty analysis

The unsteady uncertainty analysis determines the final uncertainties in the phase-averaged velocities and Reynolds stresses FS reconstruction. The discussions cover the procedures in the order of their implementation and then summarize the results including the elemental bias errors, bias and precision limits, and total uncertainties. Descriptions of the procedures first focus on the bias limit computations for the LSR curve fits followed by the FS reconstructions. Then, small-sample precision limit procedures and total uncertainty calculation is reviewed. A summary of the bias and precision limits, total uncertainties, and global convergence is given in Table 2. A summary of the elemental bias limits is given in Table A2.

A comprehensive accounting of the uncertainties in the final results should consider the uncertainties in the LSR and FS models which are caused by uncertainties in the original measured variables from equations (4)-(6) and (9), i.e., $C_{k,i}$, $(c_k c_k)_i$, $(c_m c_n)_i$, and $\gamma_i$. The original measured variables themselves are functional relations of several variables as shown in equations (4) and (9), each of which contains systematic errors. Systematic errors in the variables of equations (4) and (9) propagate through the LSR and FS models and into the harmonic amplitudes and phases, i.e., the final representation of
the phase-averaged nominal wake. The specific uncertainty analysis methodology adopted for the unsteady PIV measurements follows Coleman and Steele (1999). The methodology provides a framework for comprehensive estimation of uncertainties associated with the LSR curve fitting and is also adapted for the estimation of uncertainties associated with the FS reconstructions. The methodology does not account for errors that arise from using the wrong mathematical model for representing a dataset, therefore, care is taken to avoid over- or under fitting the phase-averaged velocities and Reynolds stresses. For the unsteady case, the instantaneous velocities and Reynolds stresses expressed by equations (4)-(6) are a function of the incident wave phase angle at the nominal wake plane. This is evident in Fig. 6g and 6h, which shows typical velocity $U_i$ and Reynolds stress $\overline{u'u'}$ response at a grid point in the outer region as a function of $\gamma$. Additionally, equations (4)-(6) are functions of the $y$ and $z$ coordinates of the measurement area with respect to model 5512. Therefore, equations (4)-(6) for $i = 1,\ldots,N_f$ are written in functional form as

$$X_i = f\{S_{k,i}, L_{obj}, L_{img}, \Delta t, U_c, \gamma, \lambda, t, f_e, y, z\}$$  \hspace{1cm} (15)

### 3.2.1. Least-squares regression bias limits.

The expression for an $n^{th}$-order LSR model representing a phase-averaged velocity or Reynolds stress $X_{LSR,j}$ is given in equation (10). For this case, the LSR model is derived from $N_f$ data pairs that are themselves computed from functional relations, i.e., equations (4) and (9). The bias limit in $X_{LSR,j}$ at any grid point is given by

$$B^2_{X_{LSR}} = \sum_{i=1}^{N_f} \sum_{j=1}^{360} \left( \frac{\partial X_{LSR,j}}{\partial S_{k,i}} \right)^2 B^2_{S_{k,i}} + \sum_{j=1}^{360} \left( \frac{\partial X_{LSR,j}}{\partial L_{obj}} \right)^2 B^2_{L_{obj}} + \sum_{j=1}^{360} \left( \frac{\partial X_{LSR,j}}{\partial L_{img}} \right)^2 B^2_{L_{img}} +$$

$$\sum_{j=1}^{360} \left( \frac{\partial X_{LSR,j}}{\partial \Delta t} \right)^2 B^2_{\Delta t} + \sum_{j=1}^{360} \left( \frac{\partial X_{LSR,j}}{\partial U_c} \right)^2 B^2_{U_c} + \sum_{j=1}^{360} \left( \frac{\partial X_{LSR,j}}{\partial \gamma} \right)^2 B^2_{\gamma} + \sum_{j=1}^{360} \left( \frac{\partial X_{LSR,j}}{\partial \lambda} \right)^2 B^2_{\lambda} +$$

$$\sum_{j=1}^{360} \left( \frac{\partial X_{LSR,j}}{\partial t} \right)^2 B^2_{t} + \sum_{j=1}^{360} \left( \frac{\partial X_{LSR,j}}{\partial f_e} \right)^2 B^2_{f_e} + \sum_{j=1}^{360} \left( \frac{\partial X_{LSR,j}}{\partial D} \right)^2 B^2_{D} +$$

$$\sum_{j=1}^{360} \left( \frac{\partial X_{LSR,j}}{\partial z} \right)^2 B^2_{z} + \sum_{j=1}^{360} \left( \frac{\partial X_{LSR,j}}{\partial y} \right)^2 B^2_{y} + \sum_{j=1}^{360} \left( \frac{\partial X_{LSR,j}}{\partial c} \right)^2 B^2_{c}$$  \hspace{1cm} (16)
Terms on the right hand side of equation (16) account for systematic uncertainties in the \((X_{LSR,j}, \gamma_j)\) data pairs. There are no correlated bias errors and there are no terms on the right hand side associated with uncertainty in \(\gamma_j\). \(S_{k,i}\) and \(t_i\) are treated with double summations since these variables are changing in time for each added PIV sample and increasing phase angle. The remaining variables are treated with a single summation over all phase angles. The derivatives are evaluated numerically with a perturbation method. The method is implemented for any variable by perturbing the nominal LSR by 1% in either the ordinate or abscissa, the LSR is recomputed, and the differences between the perturbed and nominal LSR are summed at whole phase angles from \(\gamma_i = 1^\circ,\ldots,360^\circ\). For \(S_{k,i}\) and \(t_i\), this method is also applied sequentially over all \(i = 1,\ldots,N_f\). The methods for estimating the elemental biases are described later. The bias computation in equation (16) is an intermediate step whose values are generated for use in a subsequent, final step in the total bias limit computation for the harmonic amplitudes and phases.

3.2.2. Fourier series bias limits. The expression for an \(N^{th}\)-order FS model representing a phase-averaged velocity or Reynolds stress \(X_{FS,j}\) is given in equation (11) where the FS model is derived from 360 data pairs computed from a 5\(th\)-order version of equation (10). Since the desired result is the FS reconstruction, the amplitudes and phases \(A_n\) and \(\gamma_n\) are written in functional form as

\[
A_n = f\{ X_{LSR}, f_e, y, z \} \tag{17}
\]

\[
\gamma_n = f\{ X_{LSR}, f_e, y, z \} \tag{18}
\]

The bias limit in \(A_n\) at any grid point is given below and has the same general form for \(\gamma_n\) and is applicable for both velocity and Reynolds stress

\[
B_{A_n}^2 = \sum_{j=1}^{360} \left( \frac{\partial A_n}{\partial X_{LSR,j}} \right)^2 B_{X_{LSR,j}}^2 + \left( \frac{\partial A_n}{\partial y} \right)^2 B_y^2 + \left( \frac{\partial A_n}{\partial z} \right)^2 B_z^2 \tag{19}
\]

As in equation (16), derivatives in the first term on the right hand side of equation (19) are evaluated with a perturbation method and the bias in the LSR model \(B_{X_{LSR,j}}\) is taken
from equation (16) at the $j^{th}$ phase angle. Systematic uncertainties in the FS associated with errors in the y and z coordinate positions of the measurement area are accounted for by evaluating the spatial derivatives of the variables and combining them with $B_y$ and $B_z$. To avoid accounting twice for systematic uncertainties associated with $f_e$, this term is omitted from equation (19) as the LSR analysis previously included effects of $f_e$. Bias limits are averaged over 31 grid points along the midlines of the $xz$ and $xy$ measurement areas that are chosen for the uncertainty analysis (Fig. 5 inset #1, 2). These lines are coincident with the nominal wake of model 5512 and are discussed in more detail below.

3.2.3. **Precision limits.** The precision limits of the FS results are determined with an end-to-end, multiple-test method. Ten converged steady and unsteady datasets are obtained at zone B, plane 05 ($y = -30.48$ mm; $z = -53.34$ mm) and zone D, plane 14 ($y = 0$ mm; $z = -53.34$ mm). These locations are within the boundary layer and near the steady-flow peak values of turbulent kinetic energy (Fig. 4 inset #1, 2). As stated above, only 31 points at $x = 0.935$, or 5% of the total number of grid points in each measurement area are considered in the analysis. This procedure is followed because these points are coincident with the nominal wake plane and good representatives of the total population of measurement area grid points. The converged datasets are spaced evenly in time through the course of the experiments to account for factors that influence variability of the measurements such as ambient motions in the tank water, traverse errors in the y, z coordinates, and laser-power and seeding changes. The precision limits are computed with the standard multiple-test equation

$$P_X = KS_X / \sqrt{M}$$

(20)

where $X$ represents any phase-averaged velocity or Reynolds stress $A_n$ or $\gamma_n$, $K = 2$ is the coverage factor for 95% confidence level, and $S_X$ is the standard deviation of a sample of $M = 10$ realizations of variable $X$.

3.2.4. **Total uncertainty.** The total uncertainty in $X$ is

$$U_X^2 = B_X^2 + P_X^2$$

(21)
where the interval \( \pm U_X \) contains the true value of \( X \) 95 times out of 100.

### 3.3. Uncertainty assessment results

Referring to equation (15), there are 12 elemental biases. Five \((B_{S_k,i}, B_{\text{Lobj}}, B_{\text{Limg}}, B_{\Delta t}, B_{\lambda})\) are estimated based on manufacturers specifications, whereas the other seven \((B_{U_c}, B_{\zeta I_1}, B_{D}, B_{\lambda}, B_{fe}, B_y, B_z)\) are estimated either by previous uncertainty analysis, calibration or empirical tests. \(B_{U_c}\) is estimated based on previous uncertainty analysis, as already mentioned. \(B_{\zeta I_1}\) and \(B_{fe}\) are estimated based on numerical tests to determine the minimum phase and frequency resolution of the FS and FFT subroutines, respectively. \(B_D\) is estimated from a set of three measurements between the servo wave gage needle and the center of the measurement area with a tape measure. \(B_{\lambda}\) is estimated with an independent experiment for direct measurement of \(\lambda\) using two servo wave gages separated a known distance axially in the tank. \(B_y\) and \(B_z\) are estimated with the manufacturers accuracy specifications of the \(y\)-, \(z\)-traverses and imaging tests of location markers on the hull and the free surface. Common elemental biases between the present study and Gui et al. (2001b) are identical \((B_{S_k,i}, B_{\text{Lobj}}, B_{\text{Limg}}, B_{\Delta t}, B_{\text{tit}}, B_{U_c}, B_y, B_z)\).

Table A2 lists the estimates for the elemental biases along with their percentage contributions to the total bias limit for phase-averaged velocity and Reynolds stresses 0th-harmonic amplitudes and 1st-harmonic amplitudes and phases. The largest bias for the axial and vertical velocity amplitudes is \(B_{\text{Lobj}},\) whereas the second largest is \(B_{U_c}\) and \(B_{\lambda},\) respectively. The largest and second largest biases for the transverse velocity amplitudes are \(B_{\lambda}\) and \(B_z\). For the transverse and vertical velocity amplitudes \(B_y\) also has a significant contribution. The largest and second largest biases for the Reynolds stress amplitudes are either \(B_{S_k,i}\) or \(B_{\lambda}\). The largest and second largest biases for both the velocity and Reynolds stress phase angles are \(B_z\) and \(B_y,\) respectively. The other six biases \(B_{\text{Limg}}, B_{\Delta t}, B_{\zeta I_1}, B_D, B_{\text{tit}}, B_{fe}\) are relatively small contributing \(\leq 1\%\) to the total bias limit. Large \(B_{\text{Lobj}}\) is due to accuracy of the PIV calibration. Large \(B_{U_c}\) is due to the accuracy of the carriage speed circuit. Large \(B_{\lambda}, B_z,\) and \(B_y\) are due to the large sensitivity coefficients associated with these biases. Lastly, large \(B_{S_k,i}\) is due to the accuracy of the PIV evaluation algorithm.
Table 2 summarizes the bias and precision limits as percentages of the total uncertainty along with the total uncertainty as a percentage of the dynamic range. The bias limit is largest for the axial velocity amplitudes, whereas the precision limit is largest for the transverse and vertical velocity amplitudes. However, both bias and precision limits contribute significantly. The bias limits are much larger than the precision limits for the Reynolds stress amplitudes. The precision limit amplitudes are much larger than the bias limits for both velocity and Reynolds stress phases. The weaker signal to noise ratio for the transverse and vertical velocities accounts for their larger precision limits. The total uncertainties for the velocity amplitudes and phases are of similar magnitude as the steady-flow amplitude estimates; therefore, they are considered satisfactory. The total uncertainties for the Reynolds stress amplitudes and phases are larger than their steady-flow amplitude estimates, but still considered satisfactory in view of the complexity and difficulty making such measurements. The average convergence parameter for velocity and Reynolds stress amplitudes and phases is less than its respective uncertainty for most cases, except for some of the Reynolds stress phases. The uncertainty analysis suggests that the total uncertainty can be reduced in many cases by reduction in various elemental biases and in some cases additional data acquisition.

4. RESULTS

Of primary interest is the phase-averaged velocity and Reynolds stress nominal wake measurements for surface combatant 5512 in regular head waves. However, to put the work in perspective it is useful to first review steady nominal wake, regular head wave unsteady elevation and phase-averaged velocities, unsteady forces and moments, and phase-averaged wave field for the same conditions as for the present results. The summary is also useful for those who wish to use the data for validation of simulation methods.

4.1. Steady nominal wake

Steady-flow nominal wake data is retaken as part of the project and compared with Gui et al. (2001b) steady PIV results in order to validate newly developed data acquisition and data reduction procedures. Quantitative comparisons are made including
consideration of the data uncertainties. The data is also compared with Longo and Stern (2005) 5-hole pitot tube measurements for the same facility and model and also for two other facilities using larger geometrically similar models and again with consideration to the uncertainties (Stern et al., 2000). The overall comparisons enable error and uncertainty estimates for the current ability to measure steady ship velocity fields. Another reason for retaking the data is to map a larger region of the nominal wake plane and take data for both steady and unsteady flow at the same measurement locations facilitating direct comparisons between the steady and unsteady measurements. Lastly, it is useful to review the steady flow prior to discussing the unsteady-flow results.

### 4.1.1. Error/uncertainty estimates for ship velocity fields.

As discussed in preceding sections there are many factors that affect the accuracy of steady PIV measurements. Quantitative comparisons of present $D_1$ and Gui et al. (2001b) $D_2$ measurements including consideration of uncertainty estimates enable an assessment of unaccounted for bias errors. The comparison error $E = D_1 - D_2$ and uncertainty $U_E$ are defined. For $|E| \leq U_E$, the differences are within the uncertainty of the measurement system and the data is validated at the interval $U_E$. For $|E| > U_E$, the differences are greater than the uncertainty of the measurement system indicating unaccounted for bias errors with $E$ itself a better estimate of the uncertainty in the measurements. The present and previous data are interpolated onto a common 50 × 50 grid with dimensions of $-0.0475 \leq y \leq 0$, $-0.045 \leq z \leq -0.0025$ and $E$ is evaluated at all grid points (Fig. A6), as summarized using average values in Table 2. $U$, $W$, $\bar{v}v$, and $\bar{w}w$ comparison errors are larger than $U_E$ indicating unaccounted for bias errors and that their respective $E$ intervals should be used as improved uncertainty estimates. Previous steady PIV results are compared with Longo and Stern (2005) 5-hole pitot tube data in Gui et al. (2001b): for $U$ and $W$ $|E| > U_E$ with $E = 4.2\%$ and $7.9\%$, respectively, whereas for $V$ $|E| \leq U_E = 8.9\%$. Gui et al. (2001b) shows that differences are likely due to biases from use of a pitot probe for flow with velocity gradients. IIHR 5-hole pitot tube data is compared with INSEAN and DTMB 5-hole pitot data for 5415 in Stern et al. (2000).
\(V\) and \(W\mid E\geq U_E\) with \(E = 4.1\%\) and \(7.7\%,\) respectively, whereas for \(U\mid E\leq U_E = 3.9\%\).

No clear trend is observable regarding differences between facilities, model size, or 5-hole pitot versus PIV measurement systems, i.e., the differences between each are of similar magnitudes. The overall conclusion is that steady nominal wake uncertainties are no worse than \((U, V, W, \bar{u}u, \bar{v}v, \bar{w}w, \bar{u}v, \bar{u}w) = (4.3\%, 8.9\%, 7.9\%, 5.4\%, 8.4\%, 14.8\%, 7.2\%, 6.4\%)\). Evaluation and comparison of axial vorticity indicates similar patterns between facilities and measurement systems, but even larger differences than for the mean velocities (Fig. A7).

**4.1.2. Mean velocities and Reynolds stresses.** The steady nominal wake is shown in Fig. 7. The flow pattern is explained by interactions between the hull boundary layer and sonar dome (near center plane) and after body shoulder (near mid girth) outboard rotating axial vortices, as displayed by the INSEAN mean-flow map on the fore and after body and in the near wake. For \(x = 0, 0.1, 0.2, \text{ and } 0.4\), mostly thin boundary layer development is observed along the mid girth with thickening near the free surface and keel. Near-keel axial velocity contours have an elliptical shape with long axis parallel to the center plane, which is correlated with similarly shaped axial vorticity contours. For \(x = 0.6\), there is a gradual thickening of the boundary layer. Near-keel axial velocity contours have an elliptical shape rotated such that the long axis is parallel with the hull bottom, which is again correlated with similarly shaped axial vorticity contours, but with two distinct regions of high vorticity. For \(x = 0.8\) and 0.935 (nominal wake plane) axial velocity contours are hooked shaped with a bulge in the boundary layer near \(\frac{3}{4}\) girth and a thin boundary layer near the keel, which is correlated with hooked and elliptical (long axis parallel to the calm water plane) axial vorticity contours, respectively. For \(x = 1.0, 1.1, \text{ and } 1.2\), the bulge in the axial velocity contours becomes parallel to the wake center plane.

For the nominal wake, inboard of the axial vortex center \((y, z) = (-0.02, -0.02)\) and near the center plane, high momentum fluid is transported towards the hull thinning the boundary layer, whereas outboard of the vortex center low momentum fluid is transported away from the hull thickening the boundary layer. The appearance is a bulge in the axial velocity contours. The crossplane vectors are towards the hull with \(45^\circ\) and
90° angle to the hull outboard and inboard of the bulge and vortex center. At the axial vortex center $\omega_s \approx 7$ and $U \approx 65\% U_c$. The turbulent kinetic energy and Reynolds stresses correlate with the axial velocity and vorticity contours with large values especially in the thin boundary layer near the hull bottom and center plane but also in the bulge region. Maximum values for $(\sqrt{k}, \sqrt{\overline{uu}}, \sqrt{\overline{vv}}, \sqrt{\overline{ww}})$ are 5.4%, 5.3%, 4.1%, and 3.7%$U_c$. Normal Reynolds stresses are anisotropic with $\overline{uu} > \overline{vv} > \overline{ww}$. Magnitudes and trends are consistent with a flat plate boundary layer at $z/\delta = 0.5$. At the vortex center, the normal Reynolds stresses are more isotropic, i.e., $(\sqrt{\overline{uu}}, \sqrt{\overline{vv}}, \sqrt{\overline{ww}})$ are 4.3%, 3.3%, and 3.2%$U_c$. $\overline{uv}$ is negative in regions of positive $\partial U/\partial y$ and positive in regions of negative $\partial U/\partial y$ with maximum values of $\sqrt{\overline{uv}} = 2.4\% U_c$. $\overline{uw}$ is similar but correlates with $\partial U/\partial z$ with a maximum value of $\sqrt{\overline{uw}} = 2.8\% U_c$. The mean and turbulent nominal wake flow pattern shows similarity to the boundary layer and turbulence structure in the presence of common downstream vortex pairs (Pauley and Eaton, 1988 and 1989). The similarities include thinning and thickening of the boundary layer inboard and outboard of the vortex, respectively; upwash of low momentum fluid, high turbulence outboard of the vortex; isotropic normal Reynolds stresses in the vortex core; and large Reynolds shear stresses in the upwash region.

4.2. Regular head wave elevation and velocities

The regular head wave elevations have been studied in conjunction with commissioning the plunger wave maker (Longo et al., 1998) and measurements of the forces and moments and wave pattern (Gui et al., 2001c; 2002). Longo et al. (1998) show that the regular head waves follow linear plunger wave-maker theory with nonlinear effects within the uncertainty estimates, except for extreme conditions. Gui et al. (2001c; 2002) provide updated uncertainty estimates for regular head wave amplitude (0.7% dynamic range), wave frequency (2.7% dynamic range), and encounter frequency (0.4% dynamic range), which are applicable for the present conditions.

In preparation for the present experiments, stationary and towed PIV phase-averaged regular head wave measurements are made to develop data acquisition and reduction
procedures and document the velocities, including comparisons with linear 2D progressive wave theory (Longo et al., 2004b; Appendix B). For a progressive wave traveling in the positive-x direction, the incident wave elevation and axial and vertical velocities are

$$\zeta_I(x,t) = A \cos(kx - 2\pi f \tau t)$$

(22)

$$U(x, z, t) = 1 + \frac{2 \pi f A}{U_c} e^{i\zeta} \cos(kx - 2\pi f \tau t)$$

(23)

$$W(x, z, t) = \frac{2 \pi f A}{U_c} e^{i\zeta} \sin(kx - 2\pi f \tau t)$$

(24)

Note that for the current conditions the maximum values of non-dimensional wave amplitude and velocities are 0.006 and 0.046, respectively. Admittedly, the wave amplitude and wave-induced velocities are small and only somewhat larger than their estimated uncertainties; however, as shown below by comparison with theory they are arguably well resolved.

Fig. 8 shows typical wave-elevation time histories for ten carriage runs, including both phase-aligned, full time histories (Fig. 8a) over 15 seconds and a close up of one wave length (Fig. 8b) which highlights the variability in amplitude and wave length from run to run. The general appearance is a dominant 1\textsuperscript{st}-harmonic wave from one wave period to the next. The 0\textsuperscript{th}- and 2\textsuperscript{nd}-harmonic amplitudes are about 1\% of the 1\textsuperscript{st}-harmonic amplitude. The average difference between the measured elevation and theory is 1.4\%, which is larger than the uncertainty estimated at 0.7\%. Tests to measure the wave length using two wave probes have shown an average $\overline{\lambda} = 4.654$ m which is 1.7\% longer than the dispersion relation predicts $\lambda = 4.575$ m for small-amplitude, deep-water waves. Based on the average measured wavelength and 1\textsuperscript{st}-harmonic amplitude, $A k = 0.024$.

Fig. 9 shows phase-averaged regular head wave velocities at $\frac{1}{4}$ period times in the vertical plane with the top edge of the measurement area at $z = -25.0$ mm. Average differences between experiment and theory are computed for three vertical-plane measurement areas having 682 grid points each. For the $U$ component, average
differences are 1.2%, 0.9%, and 0.8% for $U_{0T} - U_{0E}$, $U_{1T} - U_{1E}$, and $\gamma_{U_{1T}} - \gamma_{U_{1E}}$, respectively, and for the $W$ component, averaged differences are 0.1%, 0.9%, and 2.0% for $W_{0T} - W_{0E}$, $W_{1T} - W_{1E}$, and $\gamma_{W_{1T}} - \gamma_{W_{1E}}$, respectively. The average differences are all well within the noise levels of the $U$ and $W$ 0th and 1st harmonics in Table 2. Patterns in the amplitude-difference contours show generally uniform differences with some random local areas where differences in theory and experiment are increased or decreased. Patterns in the phase-difference contours show local increases and decreases in the upper right quadrant of the measurement areas for $\gamma_{U_{1}}$ and $\gamma_{W_{1}}$, respectively. These repeatable patterns may be associated with a disturbance originating from the flow past the underwater part of the PIV system.

Fig. 10 shows detailed results at a point in the vertical plane with top edge of the measurement area at $z = -25.0$ mm. The 1st-harmonic amplitude of $V$ is an order-of-magnitude smaller than that for $U$ or $W$. The 2nd harmonics for the axial and vertical velocities are two orders-of-magnitude smaller than their first harmonics. The second harmonic for the transverse velocity is an order-of-magnitude smaller than its first harmonic. Reynolds stresses responses are small, but distinct trends are apparent. Axial $\overline{uu}$ and vertical $\overline{ww}$ normal stresses show increases when magnitudes of their corresponding velocities are maximum/minimum, which indicates a second-harmonic response. In fact, the 2nd-harmonic amplitudes are larger than the first for all three normal Reynolds stresses: 105%, 152%, and 140%, respectively, for $\overline{uu}$, $\overline{vv}$, and $\overline{ww}$. The amplitude of the transverse normal Reynolds stress is relatively small compared with the axial and vertical components. Shear stresses $\overline{uv}$ and $\overline{uw}$ roughly correlate with $\overline{uu}$ and $\overline{ww}$, respectively. Although the uncertainty in the measurement of the second harmonics is expected to be large there is a clear trend of large second-harmonic response for the Reynolds stresses.

4.3. Unsteady forces and moment

Gui et al. (2001c; 2002) study resistance $C_T$ and heave $C_H$ forces and pitch moment $C_M$ for 5512 for a wide range of $Fr$, $Ak$, and $\lambda$, including the present conditions, as shown
in Fig. 11 and Table 3. Fig. 11 includes corresponding wave elevations $\zeta_I$ and running mean values and FFT for all four variables. Running means display statistical convergence for all four time histories. Dominant FFT frequencies for incident wave frequency (0.584 Hz) and the encounter frequencies of the forces and moments ($\sim$ 0.922 Hz) closely correspond with the imposed conditions. The difference between the steady and twice the $0^{\text{th}}$ harmonic is referred to as added resistance and is 9% $C_T$. The $1^{\text{st}}$ harmonic is 1.38% of the steady $C_T$ and the $2^{\text{nd}}$ harmonic is 2.7% of the $1^{\text{st}}$ harmonic. The phase angles for maximum $C_T$, $C_H$, and $C_M$ correspond to wave crests at $x = 0.1$, 0.39, and 0.003, respectively, i.e., for crests on the fore body for the resistance and pitch moment and near the mid body for the heave force. Fig. A8 shows $Fr$, $Ak$, and $\lambda$ trends relative to the present condition.

4.4. Phase-averaged wave field

Gui et al. (2001c; 2002) also study the phase-averaged wave field for the present conditions, including near (-0.2 $\leq x \leq$ 1.3; 0 $\leq y \leq$ 0.082) and far (-0.2 $\leq x \leq$ 1.3; 0.082 $\leq y \leq$ 0.392) field regions using capacitance wire and servomechanism wave gauges, respectively. The results are combined providing $0^{\text{th}}$-harmonic amplitude and $1^{\text{st}}$-harmonic amplitude and phase results. RMS near field wave elevations are also measured in the transom region. Note that the incident wave amplitude $A$ is 43% of the dynamic range of the steady wave pattern. The $0^{\text{th}}$-harmonic wave pattern is within estimated uncertainties of the previously measured steady wave pattern (Longo and Stern, 2005) displaying Kelvin-type transverse and diverging waves for a slender ship. The maximum $1^{\text{st}}$-harmonic amplitude is $1.7A$. A large amplitude crest line and low amplitude trough line diverge from the fore body shoulder ($x = 0.35$) and transom corner, respectively, with a 24.5° angle to the hull center plane. The former leads and the latter lags the incident wave by $\pi/3$. The diffraction wave is defined as the difference between the $1^{\text{st}}$-harmonic response and incident wave and has a maximum amplitude of 40% of the unsteady free surface elevations. The RMS for steady and unsteady flow has similar patterns and maximum values in a narrow region close to the center plane extending 0.1$L$ aft. Away from the center plane the RMS is larger for unsteady rather than steady flow.
4.5. Unsteady nominal wake

There are three primary effects on the boundary layer and wake due to the regular head wave. The wave-induced pressure gradients cause accelerations/decelerations of the axial and vertical velocities similarly as the regular head wave, but with the possibility of under/over shoots and phase leads/lags. In the outer region, the axial and vertical velocities are in phase and $-\pi/2$ out of phase with the wave elevation, respectively, with a magnitude of oscillations $3-4.3\% U_c$ from bottom to top of measurement area. The wave-induced particle trajectories transport fluid axially and vertically with amplitude $Ae^{kz}$ in phase with the wave elevation. The effects of axial transport are likely small, but the effects of vertical transport are significant since the transport distance of 0.01 is a large fraction of the boundary layer thickness in the nominal wake plane such that velocity oscillations are $15\% U_c$. Lastly, the wave induces a time varying $\pm2.8^\circ$ angle of attack on the sonar dome resulting in unsteady sonar dome vortices with wave length $U_c/f_c = 0.54$, as shown by complementary CFD (Carrica et al., 2005).

4.5.1. 0th harmonic and streaming. The 0th-harmonic response (Fig. A9) shows significant differences from the steady flow (Fig. 7), which is highlighted by streaming defined as the difference between twice the 0th-harmonic and steady amplitudes. Fig. 12 displays streaming for the velocity components, turbulent kinetic energy, and Reynolds stresses normalized by the steady amplitude dynamic range. The maximum amplitudes are 5-20% and 70%, respectively, which are considerably larger than the uncertainties for steady and 0th-harmonic amplitudes. Amplitudes correspond to steady streaming motion induced by the incident wave and superimposed on the steady-flow pattern. The patterns for the velocity components correlate with the steady axial vorticity. The axial velocity shows increases in the bulge region and decreases in the thin boundary layer region outboard and inboard, respectively, of the vortex center. Note that the bulge and thin boundary layer regions correspond to the up and down wash regions, respectively, mentioned earlier in discussing the similarity between the steady nominal wake and boundary layer in the presence of common downstream vortex pairs. The transverse velocity shows small increases and large decreases above and below the vortex center, whereas the vertical velocity shows increases and decreases outboard and inboard.
of the vortex center, which combine to produce a streaming axial vortex with center coincident with the steady axial vortex but with opposite (inboard) rotation (Fig. A10). The turbulent kinetic energy and Reynolds stresses show positive values in the thin boundary layer region where steady values are a maximum, except $\overline{uv}$ which shows negative values in the thin boundary layer and bulge region where steady values are negative and positive, respectively. Thus, there are three overall effects of the regular head wave on the time mean flow in comparison to the steady flow. The axial velocity is larger in the bulge region. The axial vorticity is reduced with center shifted downwards and towards the center plane $(y, z) = (-0.03, -0.015)$. The normal stresses and $\overline{uw}$ are larger and the $\overline{uv}$ Reynolds stresses is smaller positive/larger negative in the thin boundary layer/bulge regions. The increases in the Reynolds stresses are substantial and much larger than their respective uncertainties.

### 4.5.2. 1\textsuperscript{st} and 2\textsuperscript{nd} harmonics.

Figures 13 and 14 provide 1\textsuperscript{st}-harmonic amplitude and phase, respectively, for the velocity components, turbulent kinetic energy, and Reynolds stresses. The axial and vertical velocity 1\textsuperscript{st}-harmonic amplitude and phase on the outer edge of the measurement region $(y = -0.06)$ recovers the uniform stream, regular head wave values. In regions where the 1\textsuperscript{st}-harmonic amplitudes are small the phase is indeterminate; therefore, when the amplitude is less than 10\% of the maximum value the phase is set to the edge values: 0, π/3, and -π/2 for $(U, V, W)$ and −π, −π, −π/2, −π, and 0 for $(\overline{uu}, \overline{vv}, \overline{ww}, \overline{uv}, \overline{uw})$.

The axial velocity 1\textsuperscript{st}-harmonic amplitude shows very large amplitudes in the bulge and thin boundary layer regions similarly but with even larger amplitudes than the streaming. The former is in phase and the latter $-\pi$ out of phase with the incident wave. These large amplitudes are attributed to the wave-induced vertical transport of low momentum fluid downward at wave troughs and high momentum fluid upward at wave crests, which results in contraction and expansion of the boundary layer in phase with the wave in the bulge region and $-\pi$ out of phase with the wave in the thin boundary layer region.

The vertical 1\textsuperscript{st}-harmonic amplitude shows zero response under the hull near the center plane, small response in bulge and thin boundary layer region, and regular head
wave/uniform flow in the outer region indicating the hull and boundary layer has a significant damping effect on the wave-induced vertical velocities. The phase is the same as for the regular head wave vertical velocity $-\pi/2$, except in the bulge and thin boundary layer region where it is in phase with the wave.

The transverse 1st-harmonic amplitude shows zero response under the hull near the center plane (by symmetry it should be identically zero on the center plane), small response in the outer region, and fairly large (similar order wave-induced velocities) at the hull shoulder near the free surface where a finger of large values is evident. The phase is $\pi/3$. Near the steady vortex center, the phase abruptly changes from $\pi$ to $-\pi$, but the amplitudes are small. The $V_1$ response is explained by continuity with $\partial U / \partial x$ small and $\partial V / \partial y = -\partial W / \partial z$. As $W$ increases positive, $V$ moves away from the hull and center plane. Therefore, $W$ and $V$ must be somewhat in phase with each other (at least we know they would be exactly in phase with each other if the hull are infinitely long with parallel sides). In consideration of the ship coordinate system, we might expect $\pi/2$ phase for $V$ since the phase for $W$ is $-\pi/2$, i.e., equal magnitude but opposite sign. However, we observe $\pi/3$. The departure from $\pi/2$ may be due to the curvature of the hull, i.e., the three-dimensional effect of the hull.

Turbulent kinetic energy and Reynolds stresses 1st-harmonic amplitudes are two/three orders-of-magnitude smaller than the $U/(V, W)$ velocities showing largest values in the bulge and thin boundary layer regions. Although the Reynolds stresses amplitudes are less than their uncertainties their patterns are distinct and seemingly reasonable. The phase is mostly the same as for the regular head wave, except in the bulge region where it is $\pi$ out of phase.

The 2nd-harmonic amplitudes are mostly an order-of-magnitude smaller than the 1st-harmonic amplitudes showing largest values in the bulge and thin boundary layer regions (Fig. A11).

4.5.3. Fourier series-reconstructed time histories. Fig. 15 shows reconstructed time histories of the axial velocity contours with cross plane vectors at $t / T = 0, 1/4, 1/2, 3/4$ including a profile view (top row) of 5512 with superimposed wave profiles at the same $t / T$. The vectors include total FS reconstruction (middle row) and 1st harmonic (bottom
The reference vectors are color-coded to the superimposed wave profiles. The contours and vectors support conclusions from the preceding section. The large increases (expansion) and decreases (contraction) in the boundary layer thickness are in phase with the regular head wave and wave-induced axial velocities. The regular head wave-induced vertical velocities are evident in the outer region and damped in the inner region and are $-\pi/2$ out of phase with the regular head wave. Large regular head wave-induced transverse velocities are displayed at the hull shoulder and near free surface with a phase of $\pi/3$. Total FS reconstruction cross plane vectors are $45^\circ$ to the hull and oscillating clockwise after a wave crest passes $x = 0.935$ and then counterclockwise after a wave trough passes $x = 0.935$. 1st-harmonic vectors more clearly define the regular head wave-induced oscillations in $V$ and $W$ showing small values everywhere for a wave crest and trough at $x = 0.935$, large downward flow toward the center plane for decreasing wave elevation at $x = 0.935$, and large upward flow away from the center plane for increasing wave elevation at $x = 0.935$.

5. CONCLUSIONS

The present results for phase-averaged organized oscillation velocities ($U,V,W$) and random fluctuation Reynolds stresses ($\overline{uu}, \overline{vv}, \overline{ww}, \overline{uv}, \overline{uw}$) for the nominal wake of a surface ship advancing in regular head waves, but restrained from body motions, are important in documenting the unsteady ship boundary layer response. In conjunction with previous studies in the same facility using the same ship model and conditions for regular head wave elevation and velocity, unsteady forces and moment, and phase-averaged wave field provide valuable benchmark data for validation of ship-hydrodynamics simulation methods. Innovative data acquisition, reduction, and uncertainty analysis procedures are developed for the phase-averaged PIV, as part of the project. Use of towed PIV and focus on phase-averaged variables and Fourier reconstructions including uncertainty assessment is unique.

Comparisons of steady-flow results with previous steady PIV and 5-hole pitot tube measurements for the same geometry and conditions including other facilities enable an overall assessment of the current ability to measure steady ship velocity fields. The steady nominal wake is explained by interactions between the hull boundary layer and
sonar dome and after body shoulder outboard rotating axial vortices thinning and thickening the boundary layer inboard and outboard of the vortex center. The appearance is a mid-girth bulge in the axial velocity contours, cross plane vectors towards the hull, and maximum Reynolds stresses in the thin boundary layer and bulge regions. The unsteady nominal wake is explained by interactions between the hull boundary layer and axial vortices and regular head wave with three primary wave-induced effects: pressure gradients $4\% U_c$, orbital velocity transport $15\% U_c$, and unsteady sonar dome lifting wake. In the outer region, the uniform flow, incident wave velocities are recovered within the experimental uncertainties. In the inner, viscous-flow region, the boundary layer undergoes significant time-varying upward contraction and downward expansion in phase with incident wave crests and troughs, respectively. The difference between twice the 0th harmonic and steady amplitudes is referred to as streaming and shows 5-20% and 70% maximum amplitudes for the velocities and Reynolds stresses, respectively. The overall effect is larger axial velocity in the bulge region, reduced axial vorticity with center shifted downwards and towards the center plane, and larger normal stresses and $\overline{u v}$ but smaller $\overline{u w}$ Reynolds stresses. The axial velocity 1st-harmonic amplitude is large in the bulge region and small in the thin boundary layer region. The former is in phase and the latter $\pi$ out of phase with the incident wave. The vertical 1st-harmonic amplitude is small except in the outer region indicating the hull and boundary layer has a damping effect on the wave-induced vertical velocities. The phase is the same as the incident wave vertical velocity, except near the center plane where it is $\pi$ out of phase. The transverse 1st-harmonic amplitude also is small except at the hull shoulder near the free surface where a finger of large values is evident with a phase of $\pi/3$. Reynolds stress amplitudes are an order-of-magnitude smaller than the velocities showing largest values in the bulge and thin boundary layer regions with mostly the same phase as the incident wave.

The CFD Tokyo Workshop 2005 includes 5 test cases. The first is calm water bare hull resistance for the fixed condition for KCS (Test Case 1.1, 11 entries), 5415 (Test Case 1.2, 11 entries), and KVLCC2 (Test Case 1.4, 13 entries) and free to sink and trim condition for 5415 (Test Case 1.3, 1 entry). The second is calm water self-propelled condition for KCS (Test Case 2; 4 entries). The third is calm water static drift condition
for KVLCC2 (Test Case 3; 16 entries). The fourth is the incident-wave, forward-speed diffraction condition for 5415 (Test Case 4; 4 entries) for which the present data is used. In addition, grid dependence is studied for the KVLCC2 (Test Case 5, 11 entries). The CFD codes are challenged by the free to sink and trim, self-propelled, and forward-speed diffraction conditions, as indicated by so few entries. Test Case 4 does not make use of all the present data, but only includes comparisons for \(0^{th}\)-harmonic amplitude and \(1^{st}\)-harmonic amplitude and phase, resistance and heave forces, pitch moment, and phase-averaged wave field and nominal wake velocities, and in the latter cases Fourier series reconstructions at quarter period time intervals. Even partial use of the present data proves useful in evaluating the capabilities of the CFD codes for unsteady flow. The agreement is in part impressive, but achieved collectively since some codes perform better/worse for different variables.

Future experiments of the present kind surely will be even more complex as CFD matures for more realistic and smaller scale simulations. For ship hydrodynamics, motions, maneuvering, and self-propelled trajectories are of interest, including global and local variables. Even with current advancements in stereo PIV measurement systems for both free surface and velocity fields, such experiments remain a formidable resourceful challenge. Adding to this challenge are the requirements for smaller and smaller scales demanding yet higher data rates and resolution.

6. ACKNOWLEDGEMENTS

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7. REFERENCES


Table 1. Geometric parameters for DTMB model 5512 and full scale.

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Table 2. Summary of steady- and unsteady-flow PIV uncertainty assessment results.

### Steady-flow PIV UA

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†: nominal wake uncertainty results from Gui et al. 2001b; ‡: zero carriage speed test results; ‡‡: uniform flow test results

### Unsteady-flow PIV UA

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<td>0.054</td>
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<td>0.058</td>
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<td>0.021</td>
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<td>0.003</td>
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<td>81.7</td>
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†: nominal wake uncertainty results from Gui et al. 2001b; ‡: zero carriage speed test results; ‡‡: uniform flow test results
Table 3. Fourier series reconstruction of the regular head wave and forces and moment coefficients.

<table>
<thead>
<tr>
<th>Term</th>
<th>$\zeta_i$ (cm)</th>
<th>$C_T$</th>
<th>$C_H$</th>
<th>$C_M$</th>
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<tr>
<td>$f$ (Hz)</td>
<td>0.584</td>
<td>0.924</td>
<td>0.918</td>
<td>0.919</td>
</tr>
<tr>
<td>$X_{sub} \times 2$</td>
<td>-</td>
<td>0.884e-02</td>
<td>-0.633e-01</td>
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<td>$X_0$</td>
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<td>0.926e-02</td>
<td>-0.680e-01</td>
<td>-0.118e-02</td>
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<tr>
<td>$X_1$</td>
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<td>0.608e-02</td>
<td>0.356e-01</td>
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<td>$X_{add}$</td>
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<td>0.402e-03</td>
<td>-</td>
<td>-</td>
</tr>
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<td>$\gamma$</td>
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<td>-46.9</td>
<td>243.4</td>
<td>-36.7</td>
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</table>

$\zeta_i$ in crest in relation to ship model

$X_{max}$ (nd) | - | 0.100 | 0.390 | 0.003 |
$X_{x_{max}}$ (nd) | - | 0.849 | 1.140 | 0.752 |
Fig. 1. DTMB model 5512.

Fig. 2. IIHR facility and phase-averaged nominal wake experimental setup: (a) towing tank, wavemaker, DTMB model 5512, 2D PIV system, servo wave gage; and (b) Cartesian coordinate system. Towing tank is not to scale in the $x$-coordinate.
Fig. 3. Sideview schematic of phase translation to PIV measurement area.

Fig. 4. PIV measurements at the nominal wake plane of DTMB model 5512 showing both configuration #1 \(xz\) and configuration #2 \(xy\) of the 2D PIV system. Inset #1 and #2 highlight the \(xz\) and \(xy\) measurement zones, respectively, and the convergence parameter and UA precision-limit locations.
Fig. 5. Typical convergence parameter results for (a) \( U_0 \); (b) \( U_1 \); and (c) \( \gamma_{U_1} \) at zone B, plane 05 (\( y = -30.48 \text{ mm}; z = -53.34 \text{ mm} \)).
Fig. 6. Typical unsteady PIV data-reduction procedures: (a) unsorted, unfiltered data; (b) phase-sorted, unfiltered data and 1st-stage range-filter limits; (c) 1st-stage range-filtered data and 2nd-stage range-filter limits; (d) range- and median-filtered data with LSR curvefit and FS expansion of LSR curvefit; (e) mean axial velocity response and LSR and FS in the xz precision limit measurement area; (f) axial normal stress response and LSR and FS in the xz precision limit measurement area; (g) mean axial velocity response and LSR and FS in the external flow; (h) axial normal stress response and LSR and FS in the external flow.
Fig. 7. Steady-flow mean velocities and Reynolds stresses.
Fig. 8. Regular head wave elevation time histories from an upstream servo wave gage for a group of 10 unsteady PIV tests for the case with model: (a) phase-aligned full-time histories; and (b) closeup of one wavelength.

Fig. 9. Regular head wave vectors and free surface elevation at quarter periods in one encounter period for the case without model and $Ak = 0.025$, $\lambda = 4.572$ m. The vectors are colored with $U_1$: (a) $t / T = 0$; (b) $t / T = 0.25$; (c) $t / T = 0.50$; (d) $t / T = 0.75$. 
Fig. 10. Typical regular head wave flow field results at $z = -25$ mm for the case with no model and $Ak = 0.025$, $\lambda = 4.572$ m: (a) $U$; (b) $V$; (c) $W$; (d) $\overline{uu}$; (e) $\overline{vv}$; (f) $\overline{ww}$; (g) $\overline{uv}$; and (h) $\overline{uw}$.
Fig. 11. Time histories (top; solid lines) and running averages (top; dashed lines) of $\zeta_i$, $C_T$, $C_H$, and $C_M$ for $Ak = 0.025$, $\lambda = 4.572$ m, $Fr = 0.28$. FFT’s (bottom) highlight regular head wave and encounter frequencies.
Fig. 12. Differences in 0th-harmonic amplitude and steady-flow variables as a percentage of the steady variable dynamic range \([\text{streaming} = (X_0 - X_{\text{steady}})/D_X \times 100\%]\).
Fig. 13. 1st-harmonic amplitude (non dimensional) for phase-averaged velocities and Reynolds stresses.
Fig. 14. 1\textsuperscript{st}-harmonic phase (radians) for phase-averaged velocities and Reynolds stresses.
Fig. 15. FS reconstruction of unsteady nominal wake $U$ contours and $VW$ vectors at quarter periods for total magnitude (middle) and 1st harmonic (bottom): (a) $t / T = 0$; (b) $t / T = 1/4$; (c) $t / T = 1/2$; (d) $t / T = 3/4$. Reference vectors are color-coded to the regular head waves (top).
Table A1. Percent differences between PIV data modeling methods (5th-order least-squares regression versus arithmetic mean) for cases with and without model in the outer flow.

<table>
<thead>
<tr>
<th>Term</th>
<th>$U^+$</th>
<th>$V^+$</th>
<th>$W^+$</th>
<th>$\overline{uu}^+$</th>
<th>$\overline{vv}^+$</th>
<th>$\overline{ww}^+$</th>
<th>$\overline{uv}^+$</th>
<th>$\overline{uw}^+$</th>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>Without model (%)</td>
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<td></td>
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<tr>
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<td>With model (%)</td>
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†: average value from 682 measurement area grid points
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<th>$L_{img}$</th>
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<th>$U_c$</th>
<th>$\gamma_f$</th>
<th>$D$</th>
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<td>Pixel</td>
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<td>m/s</td>
<td>(°)</td>
<td>m</td>
<td>m</td>
<td>sec</td>
<td>Hz</td>
<td>mm</td>
<td>mm</td>
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<td>490</td>
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<td>-</td>
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### 0th-harmonic amplitude

| $U_0$ | 0.24 | 50.52 | 1.10 | 0.19 | 24.31 | 0.00 | 0.23 | 12.87 | 0.00 | 0.07 | 2.36 | 8.14 |
| $V_0$ | 9.83 | 1.64  | 0.04 | 0.01 | 0.79  | 0.00 | 1.11 | 60.27 | 0.16 | 0.39 | 10.30| 15.49|
| $W_0$ | 10.59| 32.87 | 0.72 | 0.12 | 15.82 | 0.00 | 0.36 | 18.51 | 0.04 | 0.17 | 6.19 | 14.61|
| $\overline{U}_0$ | 39.19| 10.05 | 0.22 | 0.04 | 4.93  | 0.00 | 0.75 | 42.38 | 0.13 | 0.34 | 1.42 | 0.57 |
| $\overline{V}_0$ | 31.55| 15.14 | 0.34 | 0.06 | 7.43  | 0.00 | 0.74 | 42.36 | 0.18 | 0.23 | 1.72 | 0.26 |
| $\overline{W}_0$ | 90.11| 1.67  | 0.04 | 0.01 | 0.82  | 0.00 | 0.13 | 6.50  | 0.06 | 0.06 | 0.33 | 0.28 |
| $\overline{U}V_0$ | 33.94| 13.49 | 0.30 | 0.05 | 6.62  | 0.00 | 0.75 | 43.90 | 0.12 | 0.21 | 0.37 | 0.25 |
| $\overline{U}W_0$ | 94.67| 1.32  | 0.03 | 0.00 | 0.65  | 0.00 | 0.06 | 3.04  | 0.02 | 0.03 | 0.07 | 0.11 |

### 1st-harmonic amplitude

| $U_1$ | 0.26 | 53.43 | 1.16 | 0.20 | 25.71 | 0.00 | 0.21 | 11.04 | 0.00 | 0.07 | 1.81 | 6.12 |
| $V_1$ | 6.78 | 1.17  | 0.03 | 0.00 | 0.56  | 0.00 | 0.74 | 38.02 | 0.11 | 0.25 | 23.19| 29.15|
| $W_1$ | 9.27 | 29.14 | 0.63 | 0.11 | 14.02 | 0.00 | 0.35 | 16.68 | 0.03 | 0.17 | 13.80| 15.80|
| $\overline{U}_1$ | 39.16| 10.66 | 0.24 | 0.04 | 5.24  | 0.00 | 0.74 | 41.46 | 0.24 | 0.33 | 0.77 | 1.25 |
| $\overline{V}_1$ | 35.28| 16.36 | 0.36 | 0.06 | 8.03  | 0.00 | 0.66 | 36.41 | 0.21 | 0.20 | 1.36 | 1.07 |
| $\overline{W}_1$ | 88.88| 1.70  | 0.04 | 0.01 | 0.84  | 0.00 | 0.15 | 7.44  | 0.05 | 0.07 | 0.23 | 0.59 |
| $\overline{U}V_1$ | 36.49| 14.10 | 0.31 | 0.05 | 6.92  | 0.00 | 0.68 | 39.26 | 0.13 | 0.18 | 1.03 | 0.83 |
| $\overline{U}W_1$ | 93.84| 1.34  | 0.03 | 0.01 | 0.66  | 0.00 | 0.07 | 3.47  | 0.02 | 0.04 | 0.19 | 0.34 |

### 1st-harmonic phase

| $\gamma_{U1}$ | 0.05 | 11.09 | 0.25 | 0.04 | 5.34  | 0.00 | 0.05 | 2.73  | 0.00 | 0.01 | 12.18| 68.30|
| $\gamma_{V1}$ | 0.51 | 0.08  | 0.00 | 0.00 | 0.04  | 0.00 | 0.07 | 3.67  | 0.01 | 0.02 | 44.06| 51.54|
| $\gamma_{W1}$ | 0.81 | 2.50  | 0.05 | 0.01 | 1.20  | 0.00 | 0.03 | 1.38  | 0.00 | 0.01 | 34.39| 59.62|
| $\gamma_{\overline{U}1}$ | 1.92 | 0.74  | 0.02 | 0.00 | 0.37  | 0.00 | 0.06 | 3.80  | 0.00 | 0.02 | 35.50| 57.57|
| $\gamma_{\overline{V}1}$ | 1.06 | 0.55  | 0.01 | 0.00 | 0.27  | 0.00 | 0.03 | 2.03  | 0.01 | 0.01 | 43.01| 53.02|
| $\gamma_{\overline{W}1}$ | 4.66 | 0.08  | 0.00 | 0.00 | 0.04  | 0.00 | 0.01 | 0.28  | 0.00 | 0.00 | 38.60| 56.32|
| $\gamma_{\overline{U}V1}$ | 1.11 | 0.45  | 0.01 | 0.00 | 0.22  | 0.00 | 0.02 | 1.55  | 0.00 | 0.00 | 46.19| 50.44|
| $\gamma_{\overline{U}W1}$ | 12.84| 0.16  | 0.00 | 0.00 | 0.08  | 0.00 | 0.00 | 0.24  | 0.00 | 0.00 | 27.35| 59.31|
Fig. A1. Block diagram of phase-averaged nominal wake data-acquisition and data-reduction procedures.
Fig. A2. Convergence parameter contours for phase-averaged velocities and Reynolds stresses: (a) $U_0$; (b) $W_0$; (c) $\overline{uu}_0$; (d) $\overline{ww}_0$; (e) $\overline{uw}_0$; (f) $U_1$; (g) $W_1$; (h) $\overline{uu}_1$; (i) $\overline{ww}_1$; (j) $\overline{uw}_1$; (k) $\gamma_{U_1}$; (l) $\gamma_{W_1}$; (m) $\gamma_{uu_1}$; (n) $\gamma_{ww_1}$; (o) $\gamma_{uw_1}$. 
Fig. A3. Convergence parameter contours for phase-averaged velocities and Reynolds stresses: (a) $U_0$; (b) $V_0$; (c) $\bar{u}u_0$; (d) $\bar{v}v_0$; (e) $\bar{u}v_0$; (f) $U_1$; (g) $V_1$; (h) $\bar{u}u_1$; (i) $\bar{v}v_1$; (j) $\bar{u}v_1$; (k) $\gamma_{U_1}$; (l) $\gamma_{V_1}$; (m) $\gamma_{\bar{u}u_1}$; (n) $\gamma_{\bar{v}v_1}$; (o) $\gamma_{\bar{u}v_1}$. 
Fig. A4. Typical data patching and filtering across zones A, B, C.

Fig. A5. Comparison of previous (Gui et al. 2001b) and current xy-configuration data at \( z = -0.025 \).
Fig. A6. Comparison of previous (Gui et al. 2001b) and current steady PIV measurements: (a) $U$; (b) $V$; (c) $W$; (d) $k$; (e) $\overline{uw}$; (f) $\overline{vv}$; (g) $\overline{ww}$; (h) $\overline{uv}$; (i) $\overline{uw}$.
Fig. A7. Steady-flow nominal wake axial vorticity for three facilities and two measurement systems: (a) DTMB (pitot); (b) INSEAN (pitot); (c) IIHR (pitot); (d) IIHR (2D PIV #1); and (e) IIHR (2D PIV #2).
Fig. A8. 0th- and 1st-harmonic amplitudes and added resistance for test cases with $Ak = 0.025$, $\lambda = 4.572$ m, $Fr$ = 0.19, 0.28, 0.34, 0.41: (a) $C_T$; (b) $C_H$; (c) $C_M$; (d) $C_{T,ad}$. 
Fig. A9. 0th-harmonic amplitude (non dimensional) for phase-averaged velocities and Reynolds stresses.
Fig. A10  Axial vorticity ($\omega_x$) at the nominal wake plane: (a) steady-flow case; (b) 0th harmonic; and (c) axial vorticity streaming.
Fig. A11. 2nd-harmonic amplitude (non dimensional) for phase-averaged velocities and Reynolds stresses.
Phase-averaged towed PIV measurements are made for regular head waves in a model-ship towing tank using both fixed and towed PIV systems in preparation for tests with a model ship. Data reduction and uncertainty assessment software are developed and tested for calm water and long-wave, low steepness conditions used in previous forces, moment, and wave pattern tests with surface combatant. The results are validated through comparisons with progressive wave theory.
1. Introduction

Focus of experimental ship hydrodynamics research is moving into unsteady viscous flows in support of unsteady Reynolds-averaged Navier Stokes (RANS) code development for simulation-based design. The forward-speed diffraction problem\(^1\), i.e., restrained body advancing in regular headwaves, is a building block problem towards ultimate goal of physical understanding and simulation of viscous nonlinear seakeeping and 6DOF maneuvering. The present study is precursory for measurement of the unsteady nominal wake of a naval combatant and provides documentation of the data-acquisition and reduction procedures along with detailed data of the incident headwave and comparisons with 2D progressive-wave theory.

2. Test Design

The tests are conducted in the 100×3×3 m IIHR tank, which is equipped with a drive carriage and trailer, wavemaker, and moveable wave dampeners. The DANTEC, towed, 2D PIV measurement system\(^2\) is linked to a servo wave gage positioned either directly over or upstream of the measurement area to phase lock the PIV measurements (Fig. 1a). The system is configurable to measure in vertical (xz; Fig. 1b) and horizontal (xy) planes. A separate measurement system is used for carriage speed. All tests are performed with \(U_c = 1.53\) m/s and without \(U_c = 0\) m/s forward speed for cases with and without waves. Headwave parameters are wavelength \(\lambda = 4.572\) m, frequency \(f_w = 0.584\) Hz, and steepness \(Ak = 0.025\) and frequency of encounter \(f_c = 0.922\) Hz, which are all based on previous studies\(^2,\text{3,4}\). Measurement area dimensions are 192×1018 pixels or 14.3×74.9 mm. PIV image pairs are taken at 133 ms intervals with \(\Delta t = 490\mu s\) between images. Data is acquired at \(z = -25.0\), -53.34, -110.45 mm and \(z = -25.0\) mm for xz and xy configurations, respectively. Datasets are acquired at each elevation and configuration through repeated carriage runs, stockpiling 1200 and 2000 vector maps for steady and unsteady cases, respectively, to obtain convergence. Data-reduction software is developed to perform harmonic data analysis and includes procedures for phase-sorting a dataset, two-stage filtering for spurious vectors, 5th-order least-squares curvefitting to filtered data, and 2nd-order Fourier series analysis of least squares. Uncertainty assessment (UA) follows standard procedures\(^5\).

3 Results and Comparison Theory

Fig. 2 illustrates experimental wave elevation and vector fields for \(z = -25.0\) mm at four instances in one encounter period (T). Expected wave-induced velocities are displayed and correlate with free surface elevation. Quantitative validation is performed through comparisons of harmonic variables of 2D progressive-wave theory and experiment. For \(z = -25.0\) mm, variables are averaged through the measurement area and experiment is subtracted from theory and expressed as a percentage. Average level of agreement for 0th- and 1st-harmonic amplitude is 0.3% and 0.8% for U,
respectively, and 0.1% and 0.8% for \( W \), respectively. For 1st-harmonic phase, agreement is 1-2% for both \( U \) and \( W \) which is roughly 3°-7°. Average turbulence is 0.01% (i.e., 0th-harmonic \( \overline{u'w'} \)), however, 1st-harmonic amplitudes of turbulence are small but significant (0.005%), near the location of the incident wave trough.

### 4. Future Work

Future work consists of steady and unsteady PIV measurements and UA of the nominal wake plane of model 5512\(^6\) for the same conditions presented herein. The data and UA will be archived at [www.iihr.uiowa.edu/~towtank](http://www.iihr.uiowa.edu/~towtank).

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### References