THE MIXING CHARACTERISTICS OF SUBMERGED
MULTIPLE-PORT DIFFUSERS FOR
HEATED EFFlUENTS IN OPEN CHANNEL FLOW

by
John R. Argue and William W. Sayre

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The Commonwealth Edison Company
and
Office of Water Resources Research

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IIHR Report No. 147
Iowa Institute of Hydraulic Research
The University of Iowa
Iowa City, Iowa

July, 1973

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ABSTRACT

The behaviour of a set of heated water jets discharging into a shallow, co-flowing environment was considered in a laboratory study; the conditions examined in the model simulated the flow of cooling water from a prototype multi-port diffuser discharging into a river or ocean current.

The relative effects of jet interaction and surface distortion were considered and three aspects of this problem singled out for particular attention: firstly, some means (quantitative) whereby stratified flow might be distinguished from mixed flow; secondly, the manner in which mixing (as measured by a coefficient of variation) varies with distance downstream from the discharge ports; thirdly, the flow regimes arising from combined mixing and buoyancy effects which can be expected in this type of flow system.

Apart from the quantities which define the ambient and jet characteristics, the port spacing distance was found to be the most significant variable. The entire program was conducted using an injection angle of 20°.

Criteria for differentiating among well-mixed flows, moderately well-mixed flows and stratified flows were determined, together with the relationship between mixing and distance. A scheme for classifying flow regimes was suggested and the published results of other studies brought
together with those of the present study. Satisfactory agreement is reported.

A procedure for "sizing" multi-port diffuser installations, using the relationships referred to above, is suggested.

ACKNOWLEDGMENTS

Except for minor revisions and additions, this report is essentially identical to the thesis submitted by the senior writer in partial fulfilment of the requirements for the degree of Master of Science in the Department of Mechanics and Hydraulics in the Graduate College of The University of Iowa. The junior writer served as research adviser.

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The service rendered by the Institute shop, under supervision of Mr. Dale C. Harris in constructing the experimental facilities is gratefully acknowledged, as is the help of Dr. P.A. Locher in the utilization of the Institute's IBM 1801 Digital Data Acquisition and Control System.
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LIST OF SYMBOLS

\( A_0 \)  individual discharge port area

\( B \)  slot jet width

\( b \)  jet radius (at distance \( s \))

\( CV \)  coefficient of (temperature) variation at a given section

\( CV_x \)  CV value at distance \( X \) from discharge point

\( CV_c \)  CV value to which mixed flow tends at large \( X \)

\( D \)  discharge port diameter

\( F \)  Froude number (various forms are used)

\( g \)  gravitational acceleration

\( H \)  ambient flow depth

\( h \)  height of port centerline above bed

\( h_o \)  cooling water flow depth (outfall to river)

\( k \)  velocity ratio (Cederwall), \( k = \frac{U_j}{U_a} \)

\( L \)  discharge port spacing distance

\( M_x \)  a measure of the "degree of mixing" at \( X \)

\( Q_a \)  total ambient flow across diffuser

\( Q_j \)  total diffuser discharge

\( q_a \)  ambient flow per unit length of diffuser

\( q_j \)  diffuser discharge per unit length

\( R \)  Reynolds number (various forms are used)

\( R_h \)  heat recovery ratio

\( R_t \)  dilution ratio
$S$  spacing parameter, $S = A_o/L^2$

$s$  distance measured along jet trajectory

$T$  computed mean temperature at a section

$T_a$  ambient flow temperature

$T_i$  temperature at the "ith" point (of a section)

$T_j$  jet flow temperature

$T_{\text{max}}$  maximum value of $T_i$ recorded at a section

$T_{\text{mix}}$  section temperature in a completely mixed flow

$U_a$  ambient flow velocity

$U_j$  jet flow velocity

$V$  volume flux ratio (Cederwall), $V = q_a/q_j$

$V_o$  cooling water flow velocity (outfall to river)

$x$  distance measured downstream from discharge point

$x_c$  distance to point where $CV = CV_c$

$x_{sd}$  distance X to commencement of confined flow

$\alpha$  coefficient of entrainment

$\theta$  jet injection angle (at discharge point)

$\nu_a$  kinematic viscosity of ambient flow

$\nu_j$  kinematic viscosity of jet flow

$\rho$  density of ambient flow

$\Delta \rho$  density difference, (ambient - jet)

$\phi$  angle of (linear) jet expansion

$\eta_c$  mixing efficiency
CHAPTER 1

INTRODUCTION

A. Pollution - "A Resource out of Place"

The problem of pollution in the environment has co-existed with mankind at least since "the city" came into existence in the major river valleys of the eastern Mediterranean and Asia some 5000 years ago. In moving thus along the road to civilisation, man exchanged the uncertainties, the hazards and the freedoms which are the lot of the nomadic food-gatherer, for the security, protection and responsibilities of community life in a fixed location. One of the responsibilities inherent in such life, involves the perennial problem of disposal of human waste - "a resource out of place".

That such waste matter is "a resource", may be gauged from the fact that the inhabitants of the prehistoric villages of Mesopotamia and elsewhere, did not dispose of human waste matter - it was too valuable a resource, used in the modest farming activities of those small communities. It is believed moreover, that both animal and human waste matter was used not only in agriculture, but also in the construction of huts (Mumford, 1961).
As these villages evolved to become the first cities, man's chances of satisfying his desire for "civilization" while living in complete harmony with his environment vanished.

Attempts by later societies to cater for the activities of civilized man within self-contained ecological systems were only successful where limitations were imposed on the number of human beings living in any one place; the colony cities of ancient Greece bear witness to this.

Of course, flowing water was used for waste disposal by all of the great river-cities of the ancient world, (a fact which, alone, possibly accounts for their seemingly unchecked growth), thereby establishing a practice which has persisted through the ages. The ancients' use of the atmosphere as a "receiver" of waste material, was however quite insignificant.

Since those times, as civilization has evolved, as population densities have increased, and as society has become more and more diverse and demanding, so too have the problems which have followed. Like its ancient counterpart, the modern metropolis uses flowing water, wherever possible, or the ocean, to receive its waste products; unlike the ancient city, however, it also discharges, daily, huge quantities of ash and gas into the atmosphere (Proceedings, Third U.S.A. National Conference on Air Pollution, 1966).

Reaction by public health authorities to the presence of
this material in the water and atmosphere has led to the concept of a maximum quantity which can be either recirculated (processed) or at least tolerated within a given system. The unenviable task of fixing the magnitude of such a quantity - a Standard - is (usually) the duty of some agency of government.

While the combined effects of the many present-day pollutants may seem to spell the certain doom of the megalopolis - major disasters (e.g. the London Fog of December, 1952) have already occurred - some far-sighted engineers believe that the technological difficulties associated with the disposal of each of these wastes, i.e., the satisfaction of all of the appropriate Standards, will be solved in the future, with one exception: the "overproduction" of heat:

"In the future, the critical limitations for urban areas will probably be the density of heat production" (Brooks, 1967a)

The concern which underlies such a statement as this stems, firstly, from the direct relationship which exists between electric power generation - the greatest producer of waste heat - and the high standard of living which is sought by every major industrial nation of the world; secondly, the far-reaching effects which waste heat can have on river, estuary and coastal ecosystems and, eventually, on man himself.
In this latter connection, it is recognized (Bregman, 1969) that the oxygen consumption of aquatic vertebrates doubles with every $10^6$ F rise in water temperature; yet a rise in water temperature in fact leads to a fall in dissolved oxygen content - the decrease being 10-15% for a $10^6$ F rise (Welch, 1969: Parker and Krenkel, 1969). The fact that it is common for power station planners to design for stream temperature increases of 5-$10^6$ F above ambient indicates the validity of the concern expressed above.

It is to this problem of waste heat dispersal in flowing environments that the present study is addressed.

B. Scope of the Study

In general, the success of any technique employed in the dispersal of waste matter may be judged on the basis of two quantities:

(i) the extent to which the pollutant is dispersed in the proximity of the discharge point, and

(ii) the concentration(s) of the pollutant within the environment encompassed by this dispersal.

It is clearly desirable that each of these quantities should be minimized; however, it is equally apparent that they cannot both take on minimum values together. And so it is that every situation involving waste dispersal must accept the
pre-dominance of one or other of these "criteria" or else compromise between them.

The aim of the present investigation was to provide some basic design information of general applicability, on the "speed" with which a row of concentrated jets of buoyant waste matter can be dispersed within the confines of a channel of flowing water. In terms of the two quantities stated above, the objective here involved obtaining minimum concentrations of pollutant dispersed evenly throughout the entire flowing environment.

Although the test program used heated water as the "waste material", the findings of the investigation apply also to some other situations where a buoyant jet is passing into a confined flowing environment, e.g., the discharge of sewage into a shallow ocean current. The form in which the results of the investigation are finally presented (Figure 6.1 involving density, not temperature) bears out this point.
CHAPTER 2
BACKGROUND LITERATURE

The extensive literature that is devoted to the general problem of waste dispersal in stagnant and flowing environments may be divided into two broad areas:-

(i) that dealing with the surface discharge of pollutants, and

(ii) that involving sub-surface discharge.

The main concern of the present investigation is with the discharge of pollutants into shallow flowing environments, an area which interfaces both of these categories; our interest in previous work must, therefore, encompass both of these modes of discharge.

A. Surface Discharge

The discharge of water-borne waste matter at or near the surface of a body of receiving water is the simplest of the commonly used dispersal techniques; the passing of sewage through an ocean outfall, the discharge of heated water from a canal into a cooling pond or river, are applications of this technique. Although the basic mechanism of contact between waste matter and receiving water is the same in each case, the
nature of the pollutant has an influence on the phenomena which are involved in the dispersal process.

In the case of sewage, dispersal is usually achieved by those mechanisms which produce dilution in fluid flow systems - turbulent diffusion, and entrainment of ambient flow into the jet. The dispersal of heated water, on the other hand, while employing the same processes of diffusion and entrainment, also involves phenomena associated with surface heat loss.

The brief resume which follows only treats this latter case - the surface discharge of heated water - since this type of discharge, as has been explained above, represents the more complex of the two practical situations.

In his very clear analysis relating to the discharge of cooling water from a power station, Harleman (1969) examined the hydraulics of an idealised system where river water was used on a "once-through" basis.

Harleman analysed the general two-layered flow system - representing the interaction of the (heated water) outflow and the colder river water - and combined a one dimensional equation of motion for the system with the continuity equation to give the single differential equation:-

$$\frac{dh_2}{dx} = \frac{\left[f_1 b_2 |V_1|V_2 - f_2 b_2 \left(\frac{h_a}{h_a - h_2}\right)(V_1 - V_2)(V_1 - V_2)\right]}{\frac{\Delta \rho}{\rho} \left[F_2^2 + F_1^2 - 1\right]}$$

... 2.1

where \( F_1 = \frac{V_1}{\sqrt{g \frac{\Delta \rho}{\rho} h_a}} \), \( F_2 = \frac{V_2}{\sqrt{g \frac{\Delta \rho}{\rho} h_2}} \),
Figure 2.1a Quantities involved in a simple stratified flow

Figure 2.1b Three possible discharge systems

Figure 2.1 Stratified Flow at a Cooling Water Outfall
By restating the interfacial shear stress terms ($T_b$, $T_i$) in terms of the corresponding dimensionless friction factors ($f_b$, $f_i$), equation 2.1 becomes:
\[
\frac{dh_{ea}}{dx} = \frac{-f_i}{\frac{\Delta \rho}{\rho} \left[ \frac{h_0}{h_e - h_{ea}} \right] V_e^2}{\frac{\Delta \rho}{\rho} \left[ \frac{h_e^2}{F_o^2 - 1} \right]}
\]
which is the general equation for interfacial slope of a two-layered stratified flow (where the lower level fluid has zero velocity).

This equation leads to a simple one dimensional criterion for predicting the behaviour of flow from a cooling water outfall channel in terms of the densimetric Froude number of the outfall channel flow (see Figure 2.1(b)):

\[
F_o = \frac{V_0}{\sqrt{g \frac{\Delta \rho}{\rho} h_0}} < 1 \quad \text{Case a}
\]
\[
F_o = 1 \quad \text{Case b}
\]
\[
F_o > 1 \quad \text{Case c}
\]

In a more recent work, Stolzenbach and Harleman (1971) have examined the jet outfall situation (for values of $F_o$ from 1 to above 15) described as "Case c" above, and have reported on its behaviour in some detail.

The relationship between densimetric Froude number and dispersion in a heated water flow system may be summarized as
follows:

<table>
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<th>( F_o = \frac{V_o}{\sqrt{g \frac{\Delta p}{\rho h_o}}} )</th>
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<th>Heat loss through surface</th>
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<tr>
<td>( F_o &lt; 1 )</td>
<td>very little</td>
<td>relatively rapid</td>
</tr>
<tr>
<td>( F_o = 1 )</td>
<td>some</td>
<td>slower</td>
</tr>
<tr>
<td>( F_o &gt; 1 )</td>
<td>relatively rapid</td>
<td>much slower*</td>
</tr>
</tbody>
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*Although, in fact, nearly all of the heat which mixes with a (receiving) body of water is ultimately lost through the surface, the time taken by this process is very long indeed; for real installations therefore, the surface heat loss which does occur in the vicinity of the outfall may be considered 'insignificant' (Brooks, 1967a).

**B. Sub-surface Discharge**

Although some mixing is undoubtedly involved in the surface discharge process - to a small extent at low \( F_o \) values, a large extent at high \( F_o \) values - extensive mixing of waste matter within a receiving body of water is more commonly
associated with sub-surface discharge from a multi-port diffuser.

Although sewage effluent has been discharged in this manner for some years, the discharge of heated water using this process is a relatively recent innovation (Brooks, 1969).

The basic unit of the multi-port diffuser is, of course, the single, submerged jet - a fluid flow system which has attracted the attention of a great many researchers during the past 25 years; and while it is true that both the behaviour and properties of the submerged jet in all "simple" environments were successfully analysed by the early years of the last decade, solutions for the more difficult cases e.g. flow into non-uniform density fields and cross flows, have only been presented in recent years. Indeed, theoretical analysis of the behaviour of jets in confined flowing environments - the realm which encompasses the present enquiry - is recognised as a very difficult problem for which only empirical solutions have, to date, been attempted (Ditmars, 1971; Harleman, Jirka and Stolzenbach, 1971; Jain, Sayre, Akyeampong, McDougall and Kennedy, 1971).

Although an early paper by Rouse (1947) on gravitational diffusion associated with plumes probably marks the commencement of work in this field, the first major contribution to the study of submerged jets is credited to Albertson, Dai, Jensen
and Rouse (1950). This investigation of momentum jet behaviour - velocity profiles, boundary expansion, relationship of volume flux and centerline velocity to distance, etc. - together with the later study of plumes by Rouse, Yih and Humphreys (1952), forms the base for all research which has been undertaken in this area.

Of equal importance in providing a theoretical approach for analysing density-differentiated jet flow systems is the integral analysis presented by Morton, Taylor and Turner (1956). This analysis involved the use of three important relationships which have been used by many researchers in later studies

(Figure 2.2a):-

(i) Conservation of volume flux

\[ \frac{dQ}{dy} = 2\pi a u_b \ldots \ldots \ldots 2.4 \]

(ii) Conservation of momentum

\[ \frac{d}{dy} \int_A \rho_{yr} \, u_{yr} \, dA = \frac{d}{dy} \left( \rho_o \frac{u_b^2}{2} \right) \]

\[ = \pi gb^2 (\rho_a - \rho_y) \ldots \ldots 2.5 \]

(iii) Conservation of density deficiency

\[ \frac{d}{dy} \int_A (\rho - \rho_{yr}) \, u_{yr} \, dA = 2\pi abu (\rho_o - \rho_a) \]

leading to:

\[ \frac{d}{dy} \left[ \pi ub^2 \left( \frac{\rho_o - \rho_y}{2} \right) \right] = \pi ub^2 \frac{d\rho_o}{dy} \ldots \ldots 2.6 \]
Figure 2.2a  
ROUND JET  $\Theta = 90^\circ$

Figure 2.2b  
SLOT JET  $0^\circ < \Theta \leq 90^\circ$

Figure 2.2  Submerged Jets
The study by Morton et al (1956), referred to above, related in particular to the behaviour of simple plumes in a linearly density-stratified environment. Morton (1959) extended this work to vertical buoyant round jets (or "forced plumes") in both uniform and linearly-stratified environments; this investigation led to a value of 0.082 for the entrainment coefficient (α in equation 2.3). Morton's name (1961) also figures in an early analysis of the problem of momentum jets and plumes in uniform co-flowing streams).

An experimental study of round jet entrainment, reported by Ricou and Spalding (1961) yielded a value of 0.057 for the momentum jet entrainment coefficient (α); this value coincided with that which is implicit in the results of Albertson et al (1950).

By 1963, these studies had disclosed the following facts relating to the behaviour and properties of momentum jets and plumes:-
Table 2.2
Momentum Jets and Plumes: Summary of Properties

<table>
<thead>
<tr>
<th>PROPERTY</th>
<th>MOMENTUM JETS</th>
<th>PLUMES</th>
</tr>
</thead>
<tbody>
<tr>
<td>boundary spread</td>
<td>( b \sim y )</td>
<td>( b \sim y )</td>
</tr>
<tr>
<td>center line velocity</td>
<td>( U \sim \frac{1}{y} )</td>
<td>( U \sim \frac{1}{\sqrt[3]{y}} )</td>
</tr>
<tr>
<td>(transverse) velocity</td>
<td>approx. Gaussian</td>
<td>approx. Gaussian</td>
</tr>
<tr>
<td>profiles</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(transverse) buoyancy</td>
<td>approx. Gaussian</td>
<td>approx. Gaussian</td>
</tr>
<tr>
<td>profiles</td>
<td></td>
<td></td>
</tr>
<tr>
<td>entrainment coefficient</td>
<td>( \alpha = 0.057 )</td>
<td>( \alpha = 0.082 )</td>
</tr>
</tbody>
</table>

However, none of these studies had tackled the case of buoyant jets discharging horizontally into stagnant environments. Abraham (1965) considered this problem (see Figure 2.2b) and applied the integral analysis to the case of a forced plume discharging \((\theta = 0)\) into a stagnant environment of uniform density. His study yielded relationships for the jet trajectory and center line concentrations. Abraham's analysis was based on an expression for jet spread which included the local value of \( \theta \) as well as distance \((s)\) along the jet trajectory. In a discussion of Abraham's paper, Fan and Brooks (1966) analyzed the problem using a linear rate of spreading, and arrived at substantially the same result.
The later study by Fan (1967) was undoubtedly the most comprehensive of all investigations up to that time. In this inquiry the integral analysis was employed together with a thorough program of experiments. The areas covered by this study included:-

(a) momentum jet into uniform-density environment,

(b) vertical buoyant jet into linearly density-stratified environment,

(c) inclined (θ > 0) buoyant jet into uniform-density environment,

(d) inclined buoyant jet into linearly density-stratified environment,

(e) vertical buoyant jet into a uniform-density ambient flow.

This project was continued by Fan and Brooks (1969) to cover the following additional areas:-

(f) round buoyant jet into uniform-density environment,

(g) round buoyant jet into density-stratified environment,

(h) slot jet into uniform and density-stratified environments,

(i) slot jet into uniform-density environment.

In the above discussion, no sharp distinction has been drawn between those which treat the submerged jet as an axisymmetric system and those which treat flow coming from a two dimensional source (or "slot jet"). Most of the fundamental studies (e.g. Albertson et al (1950), Rouse et al (1952), Abraham (1965), Fan (1967)) have in fact, treated both
cases, but have devoted more attention to round jets than to slot jets. The present inquiry, involving as it does the behaviour of jets which are initially "single" and which later merge within a confined flowing environment, must concern itself with both aspects and, in particular, the outcome of jet interaction.

This topic was the subject of a study by Koh and Fan (1970) who examined the hypothesis, widely held, that jet behaviour beyond the region where interaction takes place is indistinguishable from that of a slot jet with the same momentum flux. They considered two possible definitions of interaction:

(i) \[ 2 \sqrt{2} b = L, \] the jet spacing distance, 

where \( b = 0.114 \) s 

and \( \sqrt{2} b \) = "nominal half width".

(ii) the point where the entrainment of a round jet equals that of a slot jet.

Koh and Fan found that condition (i) occurred before condition (ii), but that the difference was not very great and that the interacting jets behaved in a manner indistinguishable from that of the line source. In this connection, Liseth (1970), made a similar study of round buoyant jets, which merged into a two-dimensional slot jet at a distance equal to 5 times the jet spacing distance.
The behaviour of buoyant slot jets in stagnant and flowing environments, was the subject of a report by Cederwall (1971). Although this investigation covered a number of aspects of slot jet behaviour in both stagnant and flowing environments - jet trajectories, dilutions, etc. - the main objective was the establishment of dimensionless criteria which could be used to predict flow regimes in practical situations.

Cederwall's study covered a wide range of flow systems, from simple momentum jets to plumes; two injection conditions were featured, horizontal and vertical. The main parameters used in this investigation were

\[ F_j = \frac{u_j^3}{\Delta \rho g q_j} \]

is the source Froude number \( \ldots \) \( 2.7 \)

and \[ V/k = \frac{u_a^2 H}{u_j^2 B} \]

is the momentum flux ratio \( \ldots \) \( 2.8 \)

where \( u_a \) = ambient flow velocity

\( u_j \) = jet flow velocity

\( H \) = ambient flow depth

\( B \) = slot (jet) width.

Three other reports of investigations into the behaviour of multi-port diffusers in confined flowing environments must be mentioned. These reports concern the model studies conducted at M.I.T. for the Brown's Ferry (Harleman, Hall and
Curtis, 1968) and Shoreham (Harleman, Jirka and Stolzenbach, 1971) Power Stations and the Iowa Institute of Hydraulic Research study for the Cordova station (Jain, Sayre, Akysampong, McDougall and Kennedy, 1971). Although each of these reports deals with specific problems, certain data — in particular, the flow regimes recorded under various flow conditions — can be adapted for use in such studies as the present inquiry.
CHAPTER 3

ANALYSIS OF THE "MIXING PROBLEM"

A. Theoretical Considerations

1. Round Jet Interference in a Shallow Flowing Environment

Consider the flow system shown in Figure 3.1a

Given that:

\[ H = \text{flow depth} \]
\[ U_a = \text{flow velocity} \]
\[ \rho = \text{density} \]
\[ v_a = \text{kinematic viscosity} \]
\[ L = \text{spacing between discharge ports} \]
\[ D = \text{discharge port diameter} \]
\[ \theta = \text{angle between port centerline and channel bed} \]
\[ h = \text{height of port centerline above channel bed} \]
\[ U_j = \text{flow velocity} \]
\[ (\rho-\Delta\rho) = \text{density} \]
\[ v_j = \text{kinematic viscosity} \]
\[ X = \text{distance downstream from discharge ports} \]
\[ M_x = \text{a measure of the "degree of mixing" at } X \]
\[ g = \text{gravitational acceleration} \]
Figure 3.1a

Figure 3.1b

Figure 3.1  Round Jets in a Flowing Environment - Terms Defined
the functional relationship which exists between the degree of mixing at a section, distant \( X \) downstream from a line of jets, and the other variables involved, may be written in the form

\[
M_X = \psi (H, U_a, \rho, \nu_a, L, D, \theta, h, U_j, (\rho - \Delta \rho), \nu_j, X, g)
\] \(3.1\)

Consider two adjacent diffuser ports which discharge into an ambient flow of depth \( H \); let the angle between the port center-line and the channel bed be \( \theta \) and the height of the center of the port discharge section be \( h \) above the channel bed; the ambient flow density is \( \rho \) and the jet flow density is \( (\rho - \Delta \rho) \) (see Figure 3.1a).

In a deep, stagnant environment, the jet centerline follows a trajectory which is **concave upwards**, a phenomenon about which much has already been written (Abraham, 1965; Fan and Brooks, 1966; Fan, 1967); where a significant ambient flow is present and the jet density is close to that of the receiving water, the centerline path of the jet is **deflected downstream**. If it is assumed that these two effects cancel one another, the jet trajectory may be approximated by a straight line - at least until it nears the flow surface - at angle \( \theta \) to horizontal (see Figure 3.1b).

In accordance with accepted submerged jet theory, consider the jet to expand linearly (angle \( \psi \)) with distance from the discharge point. At distance \( s \), the upper boundary intersects the flow surface at point \( S \); let \( b \) be the jet radius at this
section. Let \( T \) be the point on the jet centerline which is at
distance \( s \) from the port; let \( R \) be on the water surface
directly above \( T \) and \( R' \) a point directly below; \( R' \) is \( h \) above
the channel bed.

Considering the two extreme jet types - the "simple" (or
momentum) jet and the "plume" (or buoyant jet) - it may be
shown (see Table 3.1) that the predominance of jet interaction
over surface distortion (or vice versa) arises directly from the
relationship of

\[
(H-h) \quad \text{to} \quad \frac{L}{2} \left( \frac{\sin \theta}{114} + \cos \theta \right) \quad \text{for momentum jets} \quad \ldots \quad 3.2
\]

\[
(H-h) \quad \text{to} \quad \frac{L}{2} \left( \frac{\sin \theta}{099} + \cos \theta \right) \quad \text{for buoyant jets} \ldots \quad 3.3
\]

The relationship of depth \((H-h)\) to port spacing, \( L \), for two
limiting cases of interest:

\( i.e. \quad \theta = 90^\circ \quad \text{vertical jet} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad 3.4 \)

\( \theta = \phi \quad \text{flattest jet trajectory which will} \quad 3.5 \)

\( \quad \text{permit a complete submerged jet to} \)

\( \quad \text{form,} \)

is presented in Table 3.2.
## Table 3.1

Interaction and Distortion of Jets

<table>
<thead>
<tr>
<th>PROPERTY</th>
<th>MOMENTUM JET (round)</th>
<th>BUOYANT JET (round)</th>
</tr>
</thead>
<tbody>
<tr>
<td>jet radius ( b ):</td>
<td>( b = 2 ) as</td>
<td>( b = \frac{4}{5} ) as</td>
</tr>
<tr>
<td></td>
<td>where ( \alpha = 0.057 )</td>
<td>where ( \alpha = 0.082 )</td>
</tr>
<tr>
<td></td>
<td>i.e. ( b = 0.114 ) as</td>
<td>i.e. ( b = 0.099 ) as</td>
</tr>
<tr>
<td>height ( R'T ):</td>
<td>( R'T = s \sin \theta )</td>
<td>( R'T = s \sin \theta )</td>
</tr>
<tr>
<td></td>
<td>( = \frac{b \sin \theta}{0.114} )</td>
<td>( = \frac{b \sin \theta}{0.099} )</td>
</tr>
<tr>
<td>depth ( RT ):</td>
<td>( RT = b \cos \theta )</td>
<td>( RT = b \cos \theta )</td>
</tr>
<tr>
<td>depth ( H-h ):</td>
<td>( (H-h) = RT + R'T )</td>
<td>( (H-h) = RT + R'T )</td>
</tr>
<tr>
<td></td>
<td>( = b \left[ \frac{\sin \theta}{0.114} + \cos \theta \right] )</td>
<td>( = b \left[ \frac{\sin \theta}{0.099} + \cos \theta \right] )</td>
</tr>
</tbody>
</table>

Interaction between adjacent round jets will take place when

\[ 2b \geq L, \text{ the port spacing i.e. when } b \geq \frac{L}{2}, \text{ hence} \]

| Condition for jet interaction to take place before jet distortion by surface effects. | \( (H-h) > \frac{L}{2} \left[ \frac{\sin \theta}{0.114} + \cos \theta \right] \) | \( (H-h) > \frac{L}{2} \left[ \frac{\sin \theta}{0.099} + \cos \theta \right] \) |
| Condition for jet distortion by surface effects to take place before jet interaction | \( (H-h) < \frac{L}{2} \left[ \frac{\sin \theta}{0.114} + \cos \theta \right] \) | \( (H-h) < \frac{L}{2} \left[ \frac{\sin \theta}{0.099} + \cos \theta \right] \) |
### Table 3.2

Interaction and Distortion Criteria for $\theta = 90^0$ and $\phi$

<table>
<thead>
<tr>
<th>Trajectory</th>
<th>Dominance</th>
<th>Momentum Jet (round)</th>
<th>Buoyant Jet (round)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta = 90^0$</td>
<td>jet interaction</td>
<td>$H-h &gt; 4.4L$</td>
<td>$H-h &gt; 5.1L$</td>
</tr>
<tr>
<td></td>
<td>surface</td>
<td>$H-h &lt; 4.4L$</td>
<td>$H-h &lt; 5.1L$</td>
</tr>
<tr>
<td></td>
<td>distortion</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\theta = \phi$</td>
<td>jet interaction</td>
<td>$H-h &gt; L$</td>
<td>$H-h &gt; L$</td>
</tr>
<tr>
<td></td>
<td>$\phi = 6.5^0$</td>
<td></td>
<td>$\phi = 5.65^0$</td>
</tr>
<tr>
<td></td>
<td>surface</td>
<td>$H-h &lt; L$</td>
<td>$H-h &lt; L$</td>
</tr>
<tr>
<td></td>
<td>$\phi = 6.5^0$</td>
<td></td>
<td>$\phi = 5.65^0$</td>
</tr>
</tbody>
</table>

Although the original assumption used in the above analysis (straight line jet trajectory) clearly fails when applied to the limiting cases referred to above, the result is, nevertheless, revealing in that it indicates the significantly different mechanisms which come into operation for different relative magnitudes of flow depth and port spacing.

Examples of both jet interaction and surface distortion — as described above — are to be found in present-day approaches to the design of multi-port diffusers. The T.V.A. Brown's Ferry diffuser (Harleman et al, 1968) for example has the following characteristics:

$$\theta = 32^0$$

$$(H-h) = 26 \text{ ft}$$

port spacing, $L = 0.5 \text{ ft}$

hence, $$L \left( \frac{\sin \theta}{2 \cdot 114} + \cos \theta \right) = 1.375 \ldots \ldots 3.6$$
which indicates that jet interaction is undoubtedly the dominant mechanism; the design proposed for the Quad Cities diffuser (Jain et al, 1971) involves:

\[
\begin{align*}
\theta &= 20^\circ \\
(H-h) &= 10 \text{ ft} \\
L &= 40 \text{ ft} & \text{shallow section}
\end{align*}
\]

\[
\text{hence } \frac{L}{2} \left( \frac{\sin \theta}{1.14} + \cos \theta \right) = 78.8 \quad \ldots \quad \ldots \quad 3.7
\]

\[
\begin{align*}
(H-h) &= 25 \text{ ft} \\
L &= 20 \text{ ft} & \text{deep section}
\end{align*}
\]

\[
\text{hence } \frac{L}{2} \left( \frac{\sin \theta}{1.14} + \cos \theta \right) = 39.4 \quad \ldots \quad \ldots \quad 3.8
\]

which is clearly a case where surface distortion dominates.

It is apparent that considerable economies may be affected where diffusers are designed to have large port openings spaced at relatively large distances: such systems must involve mixing where surface distortion of the jet predominates over jet interaction. The experimental work of the present enquiry is drawn almost entirely from this area of the problem i.e. where heated water jets are discharged into shallow flowing environments or, simply, into open channel flows.

2. Dimensional Analysis

Before embarking upon a dimensional analysis of the "mixing problem" under consideration here, some comment must be made concerning the absence of certain variables from the list
presented in statement 3.1 above - in particular the absence of any temperature descriptors.

As has been pointed out in Chapter 1 (Section B), the mixing of heated water jets in flowing environments is only part of the much broader problem of mixing within density differentiated environments. Viewed in this light, the role of temperature in the present enquiry is seen as providing simply the means by which density differences are produced. It follows that the ambient and jet temperatures of our problem are therefore completely accounted for by the variables \( \rho, (\rho-\Delta\rho) \) respectively in statement 3.1.

From the outset of the investigation, it was envisaged that each test in the experimental program should be conducted under steady-state conditions using flow depths not less than four inches; no "time" or surface tension variables are therefore listed. The functional relationship which is sought may therefore be restated as:-

\[
\phi_1 (x, H, U_a, \rho, \nu_a, L, D, \Theta, h, U_j, (\rho-\Delta\rho), \nu_j, x, g) = 0
\]

... ... 3.9

Since 14 variables appear in the problem, it is clear that 11 dimensionless \( \pi \) terms are involved. Selecting \( H, U_j \) and \( \Delta\rho \) (where \( \Delta\rho = \) ambient flow density - jet flow density) as the primary or repeating variables, the required \( \pi \) terms in their initial and final forms - the latter obtained by permissible
combination with other \( \pi \) terms and/or dimensionless constants -
are as follows:

\[
\pi_1 = M_x \rightarrow CV_x \quad \ldots \quad \ldots \quad 3.10
\]

where \( CV_x \) is the coefficient of (temperature) variation measured
at distance \( X \) downstream from the ports:

\[
CV_x = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} (T_i - \bar{T})^2}{\frac{(\bar{T} - T_a)}{}} \quad \ldots \quad \ldots \quad 3.11
\]

where \( N = \) number of temperature readings, \( T_i \),
taken at section \( X \);

\( \bar{T} = \) mean of the \( N \) temperature readings, \( T_i \),
at section \( X \);

\( T_a = \) temperature of ambient flow.

\[
\pi_2 = \frac{U_j}{U_a} \rightarrow \frac{U_a q_a}{U_j q_j} = \frac{V}{k} \quad \ldots \quad \ldots \quad 3.12
\]

\( \frac{V}{k} \) is Cederwall's (1971) kinematic momentum flux ratio.

\[
\pi_3 = \frac{\Delta \rho}{\rho} \rightarrow \frac{U_a^3}{\frac{\Delta \rho}{\rho} q q_j} = a F_j \quad \ldots \quad \ldots \quad 3.13
\]

\( a F_j \) is the same as Cederwall's (1971) source Froude number \( F \)
where the denominator is the buoyancy flux; the subscripts "a"
and "j" are used to denote the use of ambient flow velocity in
the numerator and jet discharge in the denominator.
\[ \Pi_4 = \frac{U_H}{V_a} \quad \rightarrow \quad \frac{U_H}{V_a} = \Phi_a, \quad \ldots \quad \ldots \quad 3.14 \]

the ambient flow Reynolds number.

\[ \Pi_5 = \Theta, \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad 3.15 \]

\[ \Pi_6 = \frac{h}{H}, \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad 3.16 \]

where \( \Theta, h \) and \( H \) are as defined in Figure 3.1

\[ \Pi_7 = \frac{D}{H} \quad \rightarrow \quad \frac{\pi D^2/4}{L^2} = S, \quad \ldots \quad \ldots \quad 3.17 \]

a discharge port spacing parameter.

\[ \Pi_8 = \frac{U_jH}{\nu_j} \quad \rightarrow \quad \frac{U_jD}{\nu_j} = \Phi_j, \quad \ldots \quad \ldots \quad 3.18 \]

the jet Reynolds number.

\[ \Pi_9 = \frac{L}{H} \quad \rightarrow \quad \frac{X}{L}, \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad 3.19 \]

a measure of distance downstream from the ports in terms of port spacing distance.

\[ \Pi_{10} = \frac{H}{X} \quad \rightarrow \quad \frac{X}{H-h}, \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad 3.20 \]

another parameter which indicates downstream distance, this time in terms of ambient flow depth over the ports.

\[ \Pi_{11} = \frac{U_j^2}{gH} \quad \rightarrow \quad \frac{U_a^2}{gH} = F_a, \quad \ldots \quad \ldots \quad 3.21 \]

ambient flow Froude number.
By observing certain well established experimental procedures (Ackers, 1969), it is possible to eliminate two of these 11 dimensionless \( \pi \) terms. This is done by ensuring that the following conditions are met in each test:

\[
R_a > 600 \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad 3.22
\]

\[
R_j > 2500 \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad 3.23
\]

The ambient Flow Froude number \( (F_a) \) may also be eliminated on the grounds that the inertial forces involved in the flow system being examined are small when compared with the gravity forces (no surface disturbances of significance were observed in the tests).

Since the problem involved discharge from ports located near to the bed of a flowing environment, values of the parameter \( \frac{h}{H} \) vary only slightly enabling this term also to be dropped.

The present study arose as an extension of the Quad Cities Nuclear Power Station investigation (Jain, et al, 1971) where a jet angle, \( \Theta = 20^\circ \) was found to give the most satisfactory overall performance; this same injection angle was used throughout.

The experimental program of the study was therefore designed to determine the relationship among the remaining six dimensionless \( \pi \) terms:

\[
C V_x = \phi_2 \left( \frac{V}{k} a F_j, S, \frac{X}{L}, \frac{X}{H-h} \right) \quad \cdots \quad \cdots \quad \cdots \quad 3.24
\]
B. Investigation Objectives

Four specific objectives were sought:-

(i) to document "well mixed" and "stratified" flows downstream from a multi-port diffuser and to devise means - in terms of the pertinent flow parameters - whereby the occurrence of such flows might be predicted.

(ii) to determine how $C_{V_k}$ varies with the five quantities in 3.24.

(iii) to compare and relate qualitatively, the "types" of flows (general flow patterns, wedge forms, etc) observed in this investigation to those recorded by Cederwall in his study of slot jets in flowing environments (Cederwall, 1971).

(iv) to draw together the findings of parts (i), (ii) and (iii) above to yield a design-oriented procedure which might have application in the planning of multi-port diffuser installations.

Any attempt to distinguish between "well mixed" flow and "stratified" flow, as required by objective (i) above, must confront the basic fact that such flow descriptions are entirely qualitative and, in fact, describe only those systems
which are found at the two extremities of the general "mixed flow" spectrum.

For this reason a quantitative descriptor of "mixed flow" is required to enable "grades" of mixing to be distinguished. The parameter employed in the investigation to achieve this was the coefficient of variation \( CV_x \), whence:

"well-mixed" flows —— low values of \( CV_x \) (for large \( X \))
"stratified" flows —— high values of \( CV_x \) (for large \( X \))

Some guidance from the literature on mixed flows (Brooks, 1967b; Sayre, 1967) and "two-layered" systems (Harleman, 1961) as regards appropriate values for \( CV_x \) in the present context, led to initial adoption of the following criteria for distinguishing "well-mixed" and "stratified" flows:

**Criterion 1**

"A flow system is well-mixed if \( CV_x \leq 0.02 \)"

(the criterion finally used was \( CV_x \leq 0.03 \)).

**Criterion 2**

"A stratified flow exists when either the flow near the bed or the flow near the surface (but not both) is at ambient density."

(Although large values of \( CV_x \) — approximately 0.5 — 1.0 — undoubtedly reveal the presence of stratification, no clear-cut value can be used to define the border between those flows which are truly stratified and those which
are not).

Between these two criteria, there is a complete array of flows which are neither "well-mixed" nor "stratified" ——— flows which are referred to hereafter as "intermediate-mixed" flows.

The first aim of the project was, therefore, to differentiate flow systems into three regions ("well-mixed", "intermediate-mixed" and "stratified") of an appropriate 2-parameter plane ——— the two parameters including, between them, each of the flow quantities relevant to the phenomenon of mixing.

The second objective above relates directly to the establishment of the functional relationship posed by equation 3.1 and amplified in the dimensional analysis section of the present chapter.

The third of the stated objectives has important implications in connection with the behaviour of multi-port diffuser systems. It is generally held (Liseth, 1970; Ditmars, 1971) that, after the interaction of a row of jets has taken place, flow characteristics - velocity profiles, trajectories, dilutions, etc - all closely approximate those of an equivalent slot jet or line source.

If this is so, then the much greater versatility of the slot jet - obviating the necessity to alter port spacing, port
areas, etc - makes it a most valuable research tool.

Thus, a subsidiary aim of the present investigation, was to note certain aspects of the flow patterns associated with the mixed flows and to report their concurrence, or otherwise, with the corresponding observations by Cederwall (1971).

While the similarity of behaviour - line source and multi-port diffuser - after interaction has taken place can reasonably be conceded, similarity of performance in the present instance, where jets experience surface distortion before interaction takes place, does not follow so readily (see part A, present chapter).

To this end, Cederwall's (1971) source Froude number,

\[ F_j = \frac{U_j^3}{\Delta \rho g q_j} \]

and momentum flux ratio,

\[ \frac{V}{k} = \frac{U_j q_j}{U_j q_j} \]

were adopted as starting points (at least) for the present enquiry in order to simplify this comparative study.

As stated in Chapter 1, the overall aim of the investigation was a very practical one, namely, to provide - on the basis of a fairly generalized study - data which could be used in the design of multi-port diffuser installations;
this aim may be recognized as investigation objective (iv) above.
CHAPTER 4
EQUIPMENT AND EXPERIMENTAL PROCEDURE

A. Equipment

1. Flume and Heated Water Supply

All experiments in the test program were conducted in a 30-ft glass walled tilting flume, 24" wide, with a bed slope of 1/5000. The flume was supplied with water from a constant head tank, flow depth being controlled with the aid of an adjustable tail-gate. Heated water was discharged into this flowing environment through a set of discharge ports fitted into the floor of the flume at approximately its mid-length (see Figure 4.1).

Care was taken throughout the investigation (e.g. by suitable adjustment of discharge from each port) to ensure that a flow system was produced which simulated 2-dimensional flow as closely as possible.

Each port was made from copper pipe of internal diameter 1.05", the axis of each port where it discharged into the flow, being set at 20° to horizontal (see detail, Figure 4.2). In order to study the effect of port-spacing on the operation of such a multi-port diffuser, three different port arrangements
Figure 4.2 Discharge Ports and Thermistor Fields
were employed involving 2, 3 and 4 ports.

To ensure that the flow from each port was similar, both as regards flow rate and temperature, the heated water used in the rig was first fed into a manifold (6" diameter, 20" long) from which issued 4 pipes, each pipe being available for the supply of heated water to the (respective) diffuser ports (for 2-port tests, only two of these pipes were used, etc). The pipes were adequately insulated between the manifold and the point where they emerged from the floor of the flume.

2. Recording, Analysis and Calibration

The main task of obtaining and recording the experimental data was borne by the Institute's I.B.M.1801 Data Acquisition and Control System. This unit, which is capable of collection and on-line analysis of experimental data, was used to perform the following operations:--

(i) controlled collection of temperature data from 13 probes located in the test rig; immediate print-out and storage of this information on disk. (This sequence also included the computation and printing out of the "temperature increment": jet heated water temperature - ambient water temperature),

(ii) transfer of temperature data from disk storage to punched cards which were used to provide input data for the various processing programs run through the
University's I.B.M.360 computer,

(iii) printing out of temperature information in a concise form, i.e. data listed from punched cards.

Before the experimental program was commenced, the entire temperature collection system, comprising the thermistors at the test site and the Data Acquisition unit itself, was calibrated with the aid of an insulated box and three thermometers which could be read to 0.01°F. With this calibration completed and the appropriate calibration constants integrated into the system, a test was conducted to determine the variation in recorded thermistor values subject to a uniform temperature environment. This test showed all probes reading within a range of ± 0.02°F. Standard devices were employed to measure discharges and water depths - calibration of these devices (where necessary) being carried out in the normal manner.

B. Investigation Outlines

The complete investigation involved an introductory sampling survey and the main mixing study.

1. Sampling Survey

The aim of this phase was to determine:

(i) the duration of the sampling period which should be used in the main investigation having regard to both
accuracy and convenience, and

(ii) a suitable concentration of probes, i.e. grid system for the thermistors.

To determine a suitable sampling period, five sets of thermistor readings were taken at a certain cross section when a particular set of flow conditions had become well established. The five sets of readings corresponded to sampling periods of 20 secs., 40 secs., 60 secs., 90 secs., and 120 secs.

The second part of the sampling survey - that involving an 'acceptable' grid spacing for the thermistors - was carried out in conjunction with the early part of the main investigation itself. Sixteen sets of thermistor readings - each taken with a 21 x 9 grid pattern - were analyzed using four different spacing systems:

- Grid 1: 21 x 9
- Grid 2: 11 x 9
- Grid 3: 21 x 5
- Grid 4: 11 x 5 (see Figure 4.3)

The sixteen sets of data covered a wide range of flow systems from the "fully-mixed" condition to stratified flow and included sections taken quite close to the jet outlets (in all, seven different stable flow conditions were examined).

In both parts of the sampling survey comparison was made using $CV_x$ (see 3.11 above).
Figure 4.3 4 Grid Systems for Same Data
On the basis of these preliminary studies, the decision was made to use a sampling period of 60 seconds and the 11 x 9 grid spacing (see APPENDIX A) throughout the remainder of the investigation.

2. Mixing Investigation

2.1 Mixed and Stratified Flows

The classification of flow systems into the three categories - "well-mixed", "intermediate-mixed" and "stratified" - according to the two criteria stated in Chapter 3, Section B called for the use of two different tests:-

The Criterion 1 Test was the most generally used test in the experimental program and involved the recording of a set of temperature readings at each of a number of flow sections downstream from the discharge ports. Temperatures obtained at each of these sections (based, usually, on an 11 x 9 grid system within a field set transverse to the flow direction - see Figure 4.2) were used to compute $CV_x$, distance X signifying the position of the particular section relative to the discharge ports.

Strict application of Criterion 1 to distinguish "well-mixed" flow systems from others, involved little more than inspection of these $CV_x$ values, however it was discovered that the results of this type of test could be also most effective in distinguishing "degrees" within the range of "intermediate-
mixed" flows.

Figure 4.4 is a plot of the results of three sets of Criterion 1 Tests.

![Graph showing three cases: Case I (Test 15) with CV \( \leq 0.02 \), Case II (Test 25) with CV \( = 0.054 \), and Case III (Test 12) with CV \( = 0.07 \).]

**Figure 4.4 Typical Criterion 1 Test Results**

Case I: a "well-mixed" flow (by Criterion 1);

Case II: an "intermediate-mixed" flow for which CV has tended (i.e. at large X) towards the constant value of CV = 0.054;

Case III: an "intermediate-mixed" flow for which CV at large X has been estimated by extrapolation to tend towards CV = 0.07.

Observations such as these led to the definition of a quantity \( CV_c \) associated with the Criterion 1 Test:-

\( CV_c \): the coefficient of variation taken by a mixed flow system at large values of X (the maximum value of X being not greater than 20 times flow depth).
In accordance with this definition, the test results which are presented in Figure 4.4 appear as:

Case I: \( CV_C < 0.02 \), flow "well-mixed",
Case II: \( CV_C = 0.054 \), flow "intermediate-mixed",
Case III: \( CV_C = (0.07) \), flow "intermediate-mixed",

the brackets in Case III signifying that the value has been obtained by extrapolation.

The **Criterion 2** Test for identifying stratification involved the measurement of temperatures near the surface and near the bed of the flow; where such information was required, readings were taken at 0.05H below the free surface and 0.05H above the floor of the flume at a distance corresponding to 20 times flow depth downstream from the ports.

As mentioned, briefly, when introducing Criterion 2 (Chapter 3, Section B), classification of a flow system by comparing only surface and bed temperatures is meaningless in the context of the multi-port diffuser problem unless the measurements are taken at a point where the flow system (stratified, well-mixed, etc.) is typical of its form "at large \( X \)" (i.e. distance downstream). The standard adopted here - measurements taken where \( X = 20 \) times flow depth - was chosen mainly on grounds of flume length available downstream from the discharge ports.
It was found that, like the Criterion 1 Test, this test could be used in conjunction with a suitable experimental procedure to distinguish not only "stratified" flows from "intermediate-mixed" flows but also, the "intermediate-mixed" domain from that of the "well-mixed" flows. (The particular experimental procedure referred to here - the Comprehensive Test - is described later in the present chapter).

2.2 Mixing, Distance, Spacing and Flow

The experimental work needed to determine the inter-relationship of mixing, distance (X), port spacing and flow characteristics was encompassed by the Criterion 1 Test as described above.

2.3 Comparison with Cederwall's "Slot" Study

This part of the mixing study was qualitative in nature, and was conducted in conjunction with the Criterion 1 and Criterion 2 tests, discussed above.

It entailed such observations as the type and extent of upstream "wedge" - whenever this phenomenon was observed - as well as a general description of the downstream flow as "well-mixed", "stratified", etc., for each test.
C. Experimental Procedure

1. Planning of Experiments

Experiments in the Criterion 1 Test series were planned on the basis of Cederwall's (1971) $\frac{F_j}{a} - V/k$ plane; Figure 4.5a shows the various sets of conditions obtained by selecting appropriate values for $q_a$, $q_j$, L, H, $(T_j - T_a)$ - tested in this series.

The Criterion 2 Tests with which the Comprehensive Test is associated involved setting predetermined values of $q_a$, $q_j$, L, H to yield target values of $V/k$ (see Figure 4.5b). With any such flow established and by suitable adjustment of the jet temperature only (and hence $\Delta \rho/\rho$), it was possible to produce a sufficient range of $\frac{F_j}{a}$ values for each value of $V/k$. Temperature increments ($5^\circ F$ approx.) were employed to give the set of test points shown in Figure 4.5b.

Wherever selection of flow and/or depth was involved in the planning of tests as described above, care was taken to ensure that each of the following constraints was satisfied:

(i) ambient flow depth $> 0.395$ ft
(ii) jet Reynolds' number $> 2500$
(iii) ambient flow Reynolds' number $> 600$

When the flow, depth and temperature difference had been satisfactorily adjusted in a given test, the system was allowed
to "run" for a long period - in order to attain steady-state conditions - before any test readings were recorded. In general, this period was never less than 30 minutes and was often much longer when minor fluctuations occurred, necessitating readjustment of one or more of the components.

The next phase of the experimental procedure involved use of the IBM 1801 Data Acquisition System in conjunction with the moveable thermistor carriage.

2. Procedure

2.1 Criterion 1 Test

2.1.1 Selection of Test Sections

As described earlier, the Criterion 1 Test required temperature data to be collected at a number of sections in the field downstream from the discharge ports. The region of greatest interest for taking these cross-sections - considering the stated aims of the investigation - was down to the point where mixing appeared to cease (i.e. where coefficient of variation \( CV_c \)).

While the upstream extremity of this region was well defined, its downstream extremity was far from clear. A preliminary test was therefore conducted immediately prior to each main test to ascertain the approximate location of the section beyond which substantial mixing (as measured by the
coefficient of variation) ceased to occur.

This test - the "bed-temperature profile test" involved recording the temperature just above bed level (0.05H, approx.) at each of a number of sections - usually 8, between the discharge ports and a point 10 ft. downstream from them. A plot of these temperatures - the average of the 11 probe temperatures recorded at each section - usually took the form shown in Figure 4.6

![Diagram of Mean Bed Temperature Profile](image)

**Figure 4.6 Typical Bed Temperature Profile**

This plot gave a "first approximation" to the downstream extremity of substantial mixing, and enabled the remainder of the test program for the particular conditions to be planned (i.e. decisions relating to the location of each test cross-section).

Having selected a particular location for temperature
measurement (e.g. cross-section I), the thermistor carriage was moved into position and temperatures recorded using one of two grid systems; in the early stages of the experimental program GRID 1 (21 x 9) was used and later GRID 2 (11 x 9), (see Chapter 4, Section B (1)).

The location of the thermistor fields for the three spacing arrangements used is shown in Figure 4.2. The effluent water temperature (probe 13) and the ambient water temperature (probe 12) were recorded with each set of (carriage) "mixed flow" readings (probes 1 - 11).

2.1.2 Monitoring of Settings

Regular inspection and adjustment, when necessary, of each measuring device - flow, depth, temperature - was carried out while the experiment was being performed. Undoubtedly the most sensitive component in the system was the supply of heated water to the distribution manifold. For this reason, the printed output which followed, immediately, the collection of data by the IBM 1801, included the computation:

"TEMPERATURE INCREMENT" = (heated water temperature - ambient flow temperature.)

Thus, the temperature difference was constantly under surveillance and any serious deviations corrected immediately. Although temperature differences above 30°F were employed in
certain tests, all temperatures for accepted test runs were
within ± 0.5°F of the mean test temperature difference.

Rejection of many sets of test data did in fact occur when
a sudden, unaccounted-for fluctuation "threw" the test
temperature difference outside of the acceptable limits. In
these situations, the erroneous test run was rejected,
adjustments made to restore the conditions to normal, and the
run repeated.

2.1.3 Jet Temperature Check

Just prior to the shutting down of the system at the
conclusion of a mixing experiment, the ambient flow probe
(No. 12) was moved to the jet discharge point and the
"temperature increment" recorded for these conditions (i.e.
heated water temperature - jet temperature). The value thus
determined became a correction factor to be applied to the
(test recorded) temperature increment and thereby eliminated
two sources of error from the "raw" value, namely,
a) any temperature loss occurring between the distribution
manifold and the discharge ports, and,
b) any discrepancy in the calibration of probes 12 (ambient
flow) and 13 (heated water flow).

2.2 Criterion 2 Test - the "Comprehensive Test"

As mentioned in the previous section, the principle of the
Criterion 2 test was employed in the so-called "Comprehensive
"Test" to provide quantitative information to enable the "well-mixed", "intermediate-mixed" and "stratified" flow domains to be fixed.

Consider Harleman's (1969) use of the densimetric Froude number to distinguish "mixed flow" from "stratified flow" in the case of the side channel jet discharging into a river (see Chapter 2, Section A):

\[
P_o = \frac{V_o}{h_o} \sqrt{\frac{\Delta \rho}{\rho}} > 1 \quad \text{mixed flow} \quad \ldots \quad \ldots \quad 2.3
\]

\[
P_o = \frac{V_o}{h_o} \sqrt{\frac{\Delta \rho}{\rho}} < 1 \quad \text{stratified flow}
\]

According to this analysis, if all flow rates, velocities and depths are kept constant, then the entire spectrum of flow systems - stratified to well-mixed - can be achieved by changing only the density parameter \( \frac{\Delta \rho}{\rho} \): where this quantity is "large" flow will be stratified; where it is "small" flow will be mixed.

The Comprehensive Test utilized this hypothesis in conjunction with Criterion 2 as the basis for the following method:

i) a set of flow conditions was selected which corresponded to a certain value of \( V/k \), e.g. \( V/k \) at Figure 4.5b;

ii) the jet temperature was set at a value approximately \( 5^\circ F \) above ambient temperature. The combination of flow conditions (i) and this temperature increment specified
a particular point on the $\frac{E}{F_j} - \frac{V}{k}$ plane, e.g. the point "a" on Figure 4.5b;

iii) a temperature survey was conducted at two levels only (0.05H below surface, 0.05H above bed) at a flow cross-section approximately 20 times flow depth downstream from the discharge ports (see "Criterion 2" in Chapter 3, Section B);

iv) the jet temperature was changed (increased) by approximately $5^\circ$F - corresponding to a change to point "b" on the $\frac{E}{F_j} - \frac{V}{k}$ plane (Figure 4.5b);

v) the above procedure was continued - successive temperature increments giving rise to test conditions "c", "d", "e" ------ etc. in Figure 4.5b.

A typical set of results (surface level and bed level temperatures averaged across the grid for a wide range of jet/ ambient temperature differences) is shown in Figure 4.7.

![Figure 4.7 Typical Criterion 2 Test Result](image-url)
For each of the test points - typically a, b, c, --------- shown on Figure 4.5b, there is a corresponding pair of points - (a', a''), (b', b''), (c', c''), . . . . on Figure 4.7. The set of points a', b', c', . . . . indicates the (mean) surface temperatures recorded for the various flow and temperature conditions tested, while the points a'', b'', c'', . . . give the corresponding (mean) temperatures measured at bed level.

Where the ambient and jet temperatures are nearly the same (i.e. small values of $\Delta \rho/\rho$) the two sets of temperature readings are almost indistinguishable; however, beyond a certain point (A in Figure 4.7) they begin to diverge. The bed temperatures eventually become constant (at B in Figure 4.7) while surface temperatures continue to rise steeply as the jet temperature (and hence the ratio $\Delta \rho/\rho$) increases.

The results of this comprehensive temperature test (as presented in Figure 4.7) may be analyzed as follows:

- Flow cases $T_j - T_a < A$: well-mixed flows
- Flow cases $A < T_j - T_a < B$: intermediate-mixed flows
- Flow cases $T_j - T_a > B$: stratified flows.

The points A, B obtained from such an analysis relate directly to two points on the $F_{aj} - V/k$ plane, typically the points A' and B' on Figure 4.5b where, for $(V/k) = (V/k)_c$:

- $F_{aj}$ above A' - flow well-mixed
- $F_{aj}$ between A', B' - flow intermediate-mixed
- $F_{aj}$ below B' - flow stratified.
CHAPTER 5

RESULTS OF THE INVESTIGATION

Results of the Criterion 1 and Criterion 2 tests (see Chapter 4, Section B) were employed in the first two areas of the "mixing problem" to determine:-

(i) classification of mixed and stratified flow systems,
(ii) the inter-relationship of degree of mixing (as measured by $CV_x$, the coefficient of temperature variation) and the other variables.

Part (iii) of the investigation involved qualitative observations of flow regimes - upstream "wedge" forms, general descriptions of downstream mixing, etc - which can best be appreciated when viewed in the light of the results of parts (i) and (ii) above.

The present chapter is therefore organized along the following lines: results, analysis and discussion of parts (i) and (ii) above, followed by the results, etc of part (iii).

All experimental results arising from these investigations are presented in the Appendices.
A. Parts (i) and (ii) of the Mixing Investigation

1. Mixed and Stratified Flows

1.1 Results

In all, 32 flow systems were tested and analyzed in accordance with Criterion 1 (see Chapter 3, Section B); where appropriate, i.e. for non-"well-mixed" flows, classification on the basis of \( CV_C \) (see Chapter 4, Section B(2.1)) was also carried out with these results.

Criterion 2 tests (Chapter 4, Section B (2.1)) incorporating the Comprehensive Test (Chapter 4, Section C (2.2)) were conducted with eight different \( V/k \) (momentum flux ratio) values.

Results from both sets of tests are summarized in Figures 5.1a, b.

From an initial study of these results it may be concluded that

(i) the selection of \( CV_C \approx 0.02 \) to define a "well-mixed" flow seems quite compatible with - though not identical to - the upper limit of "intermediate-mixed" flow (as obtained from Comprehensive Tests).

(ii) the relationship of \( \alpha F_j \) to \( V/k \) along the (somewhat arbitrary) borders (W'W' and S'S') which divide the \( \alpha F_j - V/k \) plane into the three flow domains - "well-mixed", "intermediate-mixed" and "stratified" - appears
Figure 5.1a Criterion 1 Test Results

Figure 5.1b Criterion 2 Test Results

Figure 5.1 Test Results: Criterion 1 & Criterion 2 Tests
to be of the form

\[ F_j = C (V/k)^m \]

where \( m > 0 \)

within, at least, the range of values considered here.

It might be noted that the value of \( m \) is about 3 on each of the two borders.

This latter conclusion is further reinforced by the observation that the \( CV_c \) values for "intermediate-mixed" flows appear to be governed by the same basic law i.e. "intermediate-mixed" flows may be "graded" (from "stratified" to "well-mixed") with the aid of a family of lines running parallel to the domain boundaries at appropriate values of \( CV_c \) (two of these lines - \( CV_c = 0.05 \) and \( CV_c = 0.10 \) - are shown on Figure 5.1a). It should be noted that the "intermediate-mixed"/"well-mixed" boundary (W'W') approximates to a \( CV_c \) value of 0.03.

1.2 Analysis and Discussion

1.2.1 "Stratified"/"Intermediate-mixed" Boundary

Although the 'lower' set of Criterion 2 test results (around the line S'S' in Figure 5.1b) yields what appears to be a distinct boundary separating the "stratified" from the "intermediate-mixed" flows, the presence of a discharge ratio influence in the results (there is evidence of a gradation in 'V' values across the \( F_j - V/k \) plane) should be noted; unfortunately, a detailed examination of the significance of this effect is not possible in the present instance through
lack of data.

However, assuming the "stratified"/"intermediate-mixed" boundary criterion is linked with discharge ratio \( (V) \) considerations, it follows that the separation of all possible "stratified" flow cases (including those having \( V < 5 \)) from those which are clearly "intermediate-mixed" can only be achieved with an 'envelope' boundary - not a 'mean' boundary such as the line S'S' in Figure 5.1; the line SS in Figure 5.2a is therefore presented as an appropriate envelope boundary based on the present results.

The boundary SS is given by:

\[
\begin{align*}
F_j &= C_s \left( \frac{V}{k} \right)^m \\
\text{where } C_s &= 1.0 \\
m &= 3
\end{align*}
\]

1.2.2 "Intermediate-mixed"/"well-mixed" Boundary.

The 'upper' set of Criterion 2 test results (around the line W'W' in Figure 5.1b) indicates the presence of an "intermediate-mixed"/"well-mixed" interface and gives rise to a situation parallel to the case discussed above. As before, points plotted in the \( F_j - (V/k) \) plane suggest a discharge ratio \( (V) \) influence; when taken together however, these same points yield what can be regarded as a quite well-defined boundary line.

Although an 'envelope' boundary suited the
requirements of the previous case, a 'mean' boundary is considered more appropriate here on the grounds that:

i) the need to distinguish "intermediate-mixed" flows from "well-mixed" flows is not as critical in practice as the need to distinguish between flows which are stratified and those which are mixed, and,

ii) the $C_v$ values (Criterion 1 test) obtained for flow cases in the vicinity of the line $W'W'$ (on the $a_j^V/k$ plane, Figure 5.1a) all fall within the range 0.02 to 0.05; such values of $C_v$ arise from flow cases characterised by a high degree of mixing. It is not possible, using the techniques employed here, to meaningfully distinguish between such flows.

Hence, the 'mean' boundary at the "intermediate-mixed"/"well-mixed" interface - the line $W'W'$ in Figure 5.1 - becomes the basis for a criterion to distinguish between these two flow categories. This boundary is shown on Figure 5.2a as the line $WW$: 

$$a_j^F = C_W (V/k)^m$$

where $C_W = 4.0$ ...

$$m = 3$$ ...

Figure 5.2b is a compilation of information derived from the Criterion 1 and Criterion 2 tests of the present
investigation: it enables flow cases (represented by points on
the \( F_j - (V/k) \) plane) to be classified as either stratified
flows or mixed flows and, for the latter, an estimate to be
made of the degree of mixing which can be expected in any given
case.

1.2.3 A General Classification Criterion

Each of the lines presented on Figure 5.2b serves
either as a boundary enabling flow systems to be classified, or
as a guideline enabling the degree of mixing at large \( x \) to be
determined; each of these lines has the form of equation 5.1.

This observation leads to the conversion of equation 5.1 into
a single criterion for establishing both the boundaries and the
guidelines:

Equation 5.1 may be expressed as

\[
\frac{a_j}{(V/k)^m} = C \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad 5.4
\]

but

\[
\frac{F_j}{(V/k)^m} = \frac{U_j^3}{\rho g q_j} \left( \frac{U_j q_j}{a_j} \right)^m \quad \ldots \quad \ldots \quad \ldots \quad 5.5
\]

\[
= \frac{\Delta \rho}{\rho g q_j} \left( \frac{U_j}{U_j} \right)^3 \left( \frac{U_j q_j}{a_j} \right)^m \quad \ldots \quad \ldots \quad 5.6
\]

\[
= \frac{U_j^3}{a_j} \left( \frac{U_j}{U_j} \right)^{-3} \left( \frac{q_j}{a_j} \right)^m \quad \ldots \quad \ldots \quad 5.7
\]

\[
= \frac{F_j^2}{k_r} \frac{k_{m-3}}{v^m} \quad \ldots \quad \ldots \quad \ldots \quad 5.8
\]
where $F^2_j$ is the conventional jet Froude number,

$$F^2_j = \frac{u^3_j}{\frac{\Delta \rho}{\rho} g q_j} = \frac{U^2_j}{\frac{\Delta \rho}{\rho} g A_0/L}$$

... ... ... 5.9

where $A_0$ is (individual) port area,

$L$ is port spacing distance.

hence a general classification criterion may be stated:

$$F^2_j \frac{k^{m-3}}{v^m} = C$$

... ... ... ... ... 5.10

where $k = u_j/U_a$

$v = q_a/q_j$

Where $m$ and $C$ are known, this general criterion could be used in the same way as Harleman's (1969) densimetric Froude number is used to distinguish between mixed and stratified flows.

(It should, perhaps, be pointed out that although $m = 3$ has been suggested in previous sections (and is used in the applications which follow) this should not be construed as meaning either that

i) $m$ is necessarily equal to 3, or that,

ii) $m$ is necessarily the same for each criterion (stratified/mixed flow boundaries, well-mixed/intermediate-mixed flow boundary, etc).

Additional experimental work would need to be undertaken
before such statements could be made with confidence).

Assuming, on the basis of the previous sections that \( m \) takes the value 3, the general criterion for distinguishing between stratified and mixed flows becomes

\[
\frac{F_j^2}{V^3} > C_s \quad \text{for mixed flows} \quad \ldots \quad 5.11
\]

\[
\frac{F_j^2}{V^3} < C_s \quad \text{for stratified flows} \quad \ldots \quad 5.2
\]

where \( C_s = 1.0 \quad \ldots \quad \ldots \quad 5.2 \)

The corresponding criterion for distinguishing between "well-mixed" and "intermediate-mixed" flows is:

\[
\frac{F_j^2}{V^3} > C_W \quad \text{for well-mixed flows} \quad \ldots \quad 5.12
\]

\[
\frac{F_j^2}{V^3} < C_W \quad \text{for intermediate-mixed flows} \quad \ldots \quad 5.3
\]

\[
\text{where } C_W = 4.0 \quad \ldots \quad \ldots \quad 5.3
\]

2. Principal Mixing Study

The analysis described above provides a fitting background to the main mixing study which sought the relationship between degree of mixing (as measured by the coefficient of variation), distance \( \text{(X)} \), port spacing \( \text{(L)} \) and the flow parameters of shallow flowing environments.

2.1 Definition for "Shallow Flowing Environment"

To consider the particular problem of mixing in shallow flowing environments implies that attention is being focussed on those cases where surface distortion effects predominate over those of jet interaction (see Chapter 3, Section A), i.e. in terms of Table 3.1,
\[(H-h) < \frac{L}{2} \left(\frac{\sin\theta}{0.114} + \cos\theta\right)\] ... ... ... 5.13

In the present study \((\Theta = 20^\circ)\), all flow depths \((H)\) satisfied this requirement,

i.e. \(H < h + 1.97L\) ... ... ... ... 5.14

where \(h = 0.10\) ft

\(L = \) port spacing distance

While equation 5.13 provides a criterion for distinguishing between mixing systems which involve surface distortion and those which are characterised by jet interaction, the analysis must be carried a stage further to find where, in relation to the discharge ports, surface distortion in fact commences.

Assuming, as in Chapter 3, Section A, that the locus of the centerline of the discharging jet can be represented by a straight line at angle \(\Theta\) to horizontal, it follows (see Figure 5.3) that the distance \((X_{sd})\) to the point where surface distortion commences is given by

\[X_{sd} = \frac{(H-h)}{\tan(\Theta + \phi)}\] ... ... ... ... 5.15

where \(\phi = 6.5^\circ\) for momentum jets
Figure 5.3 Commencement of Surface Distortion

The distance $X_{sd}$ can be incorporated into the parameter $(X/L)_{sd}$ which gives the distance in terms of port spacing to the point where surface distortion commences in any test. Values of this parameter for the various test conditions used in the Criterion 1 tests are given in table 5.1 ($\Theta = 20^0$ in all cases).

Table 5.1
Distance to commencement of Confined Flow

<table>
<thead>
<tr>
<th>$H-h$</th>
<th>$L = 0.5$ ft (4-pipe tests)</th>
<th>$L = 0.667$ ft (3-pipe tests)</th>
<th>$L = 1.0$ ft (2-pipe tests)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.305 ft</td>
<td>$\frac{X}{L}_{sd} = 1.2$</td>
<td>$\frac{X}{L}_{sd} = 0.9$</td>
<td>$\frac{X}{L}_{sd} = 0.6$</td>
</tr>
<tr>
<td>0.710 ft</td>
<td>$\frac{X}{L}_{sd} = 2.8$</td>
<td>$\frac{X}{L}_{sd} = 2.1$</td>
<td>$\frac{X}{L}_{sd} = 1.4^*$</td>
</tr>
</tbody>
</table>

*no tests conducted in this category
The results (see APPENDIX A) reveal that in all but three tests \((x, 13, 14, 15)\), the \(CV_x\) values were drawn entirely from the region where mixing was affected by surface distortion of the jet i.e. downstream from \((X/L)_{sd}\); tests 13 and 14 include one section (each) taken upstream of this point (according to Table 5.1) and test number 15 has two (possibly three) of its four sections in this region. On these grounds, test number 15 was considered apart from the other results.

The discussion above relating to the fixing of a particular distance i.e. \((X/L)_{sd}\) in the flow field downstream from the discharge ports, leads to a definition for "shallow flowing environment":

a shallow flowing environment is the region of a flow field which is downstream from the section where distance \(x_{sd}\) is given by equation 5.15.

(distance \(x_{sd}\) is itself measured downstream from some submerged, co-flowing discharge port(s)).

The results of the present inquiry are seen therefore to come almost exclusively from such a region.

2.2 Analysis of Criterion 1 Test Results

The complex nature of the mixing processes present in a shallow flowing environment makes theoretical analysis of such a system extremely difficult - no attempt at such an analysis is presented here; instead, the required relationship (between
degree of mixing at X i.e. $C V_x$, and the other variables) was sought by entirely empirical means.

The Criterion 1 test results (see APPENDIX A) gave the (plotted) forms shown, typically, in Figure 4.4. Examination of these results gave rise to the hypothesis that the inter-relationship of $C V_x$ and X might yield a straight line when plotted as

$$\log (\psi(\text{CV}_x)) \text{ vs } \log (\psi(X)) \ldots \ldots \ldots \ 5.16$$

Plots were made, therefore, of

$$\log (\text{CV}_x) \text{ vs } \log (X/(H-h)) \ldots \ldots \ldots \ 5.17$$

and

$$\log (\text{CV}_x) \text{ vs } \log (X/L) \ldots \ldots \ldots \ 5.18$$

each of these parameters being outcomes of the dimensional analysis (Chapter 3, Section A).

These plots indicated a strong "port-spacing influence" in the ordinate which was largely accounted for by creation of the parameter $C V_x L/(H-h)$; hence adoption of the plane

$$\log C V_x \left(\frac{L}{H-h}\right) \text{ vs } \log \left(\frac{X}{L}\right) \ldots \ldots \ldots \ 5.19$$

Each of the three pipe arrangements used (2-pipes, 3-pipes, 4-pipes) gave rise to a set of lines for flows from the "well-mixed" and "intermediate-mixed" domains (see APPENDICES A and B for complete results).
If \[ X = \left[ \frac{X}{L} \right] \]

and \[ Y = CV_x \left[ \frac{L}{H-h} \right] \]

then the equations of these lines are of the form:

\[ Y = AX^N \quad \text{where } N < 0 \]

where \( A, N \) are both functions of the flow parameters and spacing, i.e.

\[ A, N = f \text{ (flow parameters, port spacing)} \]

2.2.1 Physical Significance of "A" and "N"

Before proceeding with the analysis it is important to understand the physical significance of the quantities \( A \) and \( N \).

Consider the "mixing history" of the three distinct flow types - "stratified", "intermediate-mixed", "well-mixed" - at various sections downstream from the discharge ports:

![Diagram of flow patterns](image)

**Figure 5.4 Flow Patterns - Mixed and Stratified Flows**
"Stratified" flow tends to stabilize rapidly so that the value of $CV_x$ at each test section shows little change (typical value, $CV_x = 1$; see test numbers 1, 2, 4).

"Intermediate-mixed" flow: the buoyancy and inclination of the discharging jet combine to govern early behaviour; as mixing proceeds, the lower boundary of the heated water descends to the channel bed. The $CV_x$ values cover a wide range reflecting stratified flow (upstream sections) and mixed flow (downstream sections).

"Well-mixed" flow is influenced by jet buoyancy and inclination for a short distance only and tends to stabilize rapidly; $CV_x$ values show less change than for "intermediate-mixed" flows.

Now "A" is the value of $CV_x \cdot (L/(H-h))$ at the point where $(X/L) = 1.0$, obtained (usually by extrapolation) from the $CV_x$ values measured downstream from $(X/L)_{sd}$. (Thus "A" is usually not the true value of $CV_x$ at $(X/L) = 1.0$; in certain cases where $(X/L) = 1.0$ occurs downstream from $(X/L)_{sd}$, the true value should coincide with that obtained from the tests. See Table 5.1). The magnitude of $N$ is a measure of the "rate of mixing" (i.e. the rate at which $CV_x$ is decreasing) taking place within the open channel flow.
Relating these two quantities, now, to the phenomenological descriptions presented above, the three types of flows reflect the following tendencies concerning their values of A and N:

"stratified" and "near-stratified" mixed flow:

moderately high value of A and low value of N.

"intermediate-mixed" flow: high value of A - particularly where extrapolation from downstream values of \( CV_x \) is involved - and large value of N.

"well-mixed" flow: low value of A and low value of N.

The graph presenting the 3-pipe test results (Figure B-2, APPENDIX B) demonstrates these tendencies quite clearly.

However, this description of a "well-mixed" flow in terms of the quantities A and N does not coincide with the criterion derived in Section A (1.2.3) of the present chapter: many sets of results plotted on the \( CV_x \cdot (L/(H-h)) - (X/L) \) plane, representing "well-mixed" flows according to the equation 5.12 criterion, are indistinguishable from "intermediate-mixed" flow cases so classified by the same criterion.

Since the equation 5.12 criterion for a "well-mixed" flow was fixed on a completely arbitrary though very practical basis (flows for which \( CV_x \) equals 0.03 or less are "well-mixed") it seems unwise to interfere with it; instead, another class of flow, having the properties (in terms of A and N) stated above
under "well-mixed" flow will be recognized:-

"fully-mixed" flow: a confined, mixed flow which is represented on the $CV_x = (L/(H-h)) - (X/L)\) plane by a line having low values of both $A$ and $N$ compared with the corresponding values for flows in the "intermediate-mixed" to "well-mixed" range. (It should be noted that this type of flow is described in Figure 5.5 as a "well-mixed" flow case).

Presuming that this criterion can also be described by the general classification criterion (equation 5.10) and that $m = 3$, "fully-mixed" flow may be recognized as

$$\frac{F_j^2}{V^3} > C_F \quad \ldots \ldots \ldots \quad 5.23$$

where $C_F \gg 4$

(see equation 5.30)

2.2.2 Relationship of $A$ and $N$ to Spacing and Flow Parameters

The relationship of $N$ and of $A$ to spacing distance ($L$) as represented by the spacing parameter $S = A_0/L^2$ was examined. In Figure 5.5, two sets of points are plotted to throw light on the "orders" of the types of relationships involved.

(i) average values of $-N$ and

(ii) average values of $A$

for all of the mixed flow cases which (according to their form
Figure 5.5 Relationship of A, N to Spacing Parameter S
on the $C V \times (L/(H-h)) \times (X/L)$ plane show neither the effects of stratification nor the properties of flows which are "fully-mixed".

The following relationships between $N$, $A$ and the spacing parameter $(S)$ are suggested by Figure 5.5:

$$-N = 0.83 \; S^{-1/4} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad 5.24$$

$$A = 0.068 \; S^{-1} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad 5.25$$

These empirical relationships lead to the formation of the two parameters:

$$(-NS^{1/4}) \; \text{and} \; (AS)$$

and a modified form for equation 5.22:

$$(-NS^{1/4}) = f_1 \; \text{(flow parameters)} \quad \ldots \quad \ldots \quad 5.26$$

$$(AS) = f_2 \; \text{(flow parameters)} \quad \ldots \quad \ldots \quad 5.27$$

Many trials were made to determine these relationships using the various flow parameters singly and in combinations; the most promising of these are shown in Figures 5.6 and 5.7 involving:

$$(-NS^{1/4}) = g_1 \; (q_j/q_a) = g_1 \; (1/V) \quad \ldots \quad \ldots \quad 5.28$$

$$AS = g_2 \; ((U_a/U_j)^2) = g_2 \; ((1/k)^2) \quad \ldots \quad \ldots \quad 5.29$$

In the discussion, earlier, of the physical significance of $A$ and $N$, three different types of mixed flows were recognized "intermediate-mixed" and "well-mixed" (arbitrarily separated)
and "fully-mixed"; the recognition of these categories among the flow systems used to produce Figures 5.6 and 5.7 provides the key to understanding the relationship which is sought.

While the "intermediate-mixed"/"well-mixed" boundary criterion has been established in a previous section as

\[ \frac{F_j^2}{V^3} > 4 \quad \text{for well-mixed flows} \quad \text{.. 5.3,12} \]
\[ \frac{F_j^2}{V^3} < 4 \quad \text{for intermediate-mixed flows} \]

no such criterion (other than the form suggested in equation 5.23) has been forwarded to define flows which are "fully-mixed"; however the flows shown as "fully-mixed" in Figures 5.6 and 5.7 have been so classified using the criterion:-

\[ \frac{F_j^2}{V^3} > C_F \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad 5.30 \]

where \( C_F = 15 \)

2.2.3 Relationship of \( N \) to flow parameters (see Figure 5.6)

Since the distinction drawn between the "intermediate-mixed" and "well-mixed" flows is an arbitrary one (involving \( C_{V_c} = 0.03 \)), points representing both of these classes of flow are described on the \( \left( \frac{-NS^4}{V} \right) - V \) plane (not surprisingly) by a single relationship:-

"Intermediate-mixed" and "well-mixed" flows:-

\[ \frac{-NS^4}{V} = 5/V \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad 5.31 \]

The "fully-mixed" flows i.e. flows for which \( \frac{F_j^2}{V^3} > 15 \), however, depart from this relationship and tend
Figure 5.6 \((-NS^4) - V\) Relationship
to a constant value for the parameter \((-\text{NS}^4\)) for values of \(V < 5\):

"Fully-mixed" flows:

\[-\text{NS}^4 = 0.76 \quad \ldots \quad \ldots \quad \ldots \quad 5.32\]

for \(V < 5\)

2.2.4 Relationship of \(A\) to flow parameters (see Figure 5.7)

The complicated appearance of the \(AS - k^2\) plane (compared with the plane just discussed) stems mainly from the fact that the "intermediate-mixed" - "well-mixed" flow domain is spread across a sector of the plane; its lower extremity \((S"S")\) is fixed by a stratified/mixed flow boundary and its upper limit, by a line whose position was governed by the majority of "well-mixed" flow results and the results of the "fully-mixed" flow tests. The equation of this upper boundary is

\[AS = \frac{1}{6} (10/k)^6 \quad \ldots \quad \ldots \quad 5.33\]

the curve \(W"W"\) corresponds to the (arbitrary) division drawn between "intermediate-mixed" and "well-mixed" flows (i.e. flow systems having \(CV_c = 0.03\)).

The relationship of \(AS\) to \(1/k^2\) for "fully-mixed" flows - aligned with equation 5.33 for low values of \((U_a/U_j)^2\) and "swinging" into the "intermediate-mixed" domain for high values - is in general agreement with the phenomenological explanation and definition stated (for this type of flow) in
Figure 5.7 AS - k^2 Relationship
Section A (2.2.1) of the present chapter.

3. Concluding Remarks - Parts (i) and (ii)

The aim of this section was to determine criteria for classifying the various types of stratified and mixed flows and to determine the inter-relationship of degree of mixing to the other variables involved - downstream distance, port spacing and the flow parameters. Figures 5.6 and 5.7, taken together with the various classification criteria stated in the section, constitute the successful attainment of this goal within the limitations imposed by the data (range of experimental values, etc.) used.

Application of this information to design and other situations is taken up in the concluding chapter of the report.

B. Part (iii) of the Mixing Investigation - Flow Patterns

The aim of this section was to report qualitative observations of flow patterns associated with the mixing of (round) heated water jets - upstream wedge forms and downstream behaviour in particular - and to compare these with the observations made by Cederwall (1971) in his study of slot jets.

Only a fraction of the field covered by Cederwall was examined in the present inquiry, however, the region selected - that corresponding to typical heated water discharge situations - has been studied in some detail.
Figure 5.8 Flow Regimes: Collected Data

FLOW REGIMES

ZONE A1
u/s wedge; o/s flow: stratified

ZONE A2
u/s wedge; o/s flow: mixed

ZONE B2
u/s wedge; inlet; o/s flow: mixed

ZONE C1
u/s wedge; none; o/s flow: stratified

ZONE C2
u/s wedge; none; o/s flow: mixed

KEY TO DATA
Test numbers, present study: shown — 24
Test numbers, other studies: shown —
Cedrowall (1971) — 24
Harleman et al (1970) — 21
Jain et al (1971) — 10
Goetz et al (1972) — 8

See Appendix A for details.
The observations made together with those of Cederwall are presented in APPENDIX A: a generalized summary is given in Figure 5.8.

As is to be expected, extreme "types" of flow patterns are easily distinguished, e.g., whether a distinct upstream wedge does or does not exist, whether flow is stratified or mixed (quite apart from the degree of mixing), etc. However, difficulties of interpretation must arise when any attempt is made to sub-divide the area intermediate between these two extremes without the use of some quantitative yardstick.

For this reason, flow patterns in this study were classified using 3 possible upstream forms and 2 possible downstream forms. These forms are sketched in Figure 5.9.

**UPSTREAM FORMS**

- **FORM A**
  - upstream wedge (subcritical) *descriptive term used by Cederwall (1871)*

- **FORM B**
  - incipient wedge (critical) *

- **FORM C**
  - no wedge (supercritical) *

**DOWNSTREAM FORMS**

- **FORM 1**
  - stratified flow

- **FORM 2**
  - mixed flow

Figure 5.9 Flow Regimes: Upstream & Downstream Forms
There are, of course, six possible combinations of these forms, five of which were observed in this study.

In general, the same flow pattern "types" were observed in corresponding regions of the $F_j - V/k$ plane as were reported by Cederwall. However, any attempt to make a more exacting comparison than this is met with lack of data. Of the two injection angles treated by Cederwall, $\theta = 0^\circ$ must be regarded as being closer to the present case; of the experimental results presented, only four happen to coincide with the field covered by the present study.

It is interesting to note, however, that each of these four points falling as they do about the border between zones C2 and B2, fall into the same classifications in both sets of results.

Other evidence concerning the flow patterns associated with the discharge of heated water from multi-port diffusers, comes from the Harleman, et al (1968) study for the Brown's Ferry diffuser and from the Iowa Institute of Hydraulic Research study for the proposed Cordova diffuser (Jain, et al, 1971). These studies are most helpful in providing information on upstream wedge forms: also, it is clear from the data presented in both reports that the downstream flow is at least "well-mixed" (i.e. $CV_c < 0.03$).

Another source of independent evidence is a program of student experiments (Goetz, Roman and Cadenas, 1972) carried
out at the University of Iowa to verify the existence of stratification in zone C1 (where values of \( F_j \gg 1 \)). No observations of wedge forms were reported.

The flow pattern description for each of these independent cases has been included in APPENDIX A and the points to which they correspond on the \( F_j - V/k \) plane, also presented in Figure 5.8.

There exists only one point of disagreement amongst all of this collected data – the flow regime in the vicinity of:

\[
F_j = 0.56 , \ V/k = 0.21
\]

Harleman et al (1968) show an incipient wedge in the Brown's Ferry results; Jain et al (1971) report a significant upstream wedge in the Cordova model study. Whether this discrepancy is accounted for by the different jet centerline angles used (\( \theta = 32^\circ \) in the Brown's Ferry study; \( \theta = 20^\circ \) for the Cordova model) can only be answered by more intensive research in this particular region.

However, the fact that so much agreement can be recorded for the flow regime zones in all other areas of the \( F_j - V/k \) plane allows Figure 5.8 to be put forward with some confidence.
C. Some Additional Mixing Descriptors

Almost from the outset of the present inquiry (see Chapter 3, Section A (2.1)), coefficient of variation (CV) has been employed to describe the "degree of mixing" which obtains from section to section in the flow field downstream from a multi-port diffuser. The reason for this is the almost universal place which this parameter enjoys as a descriptor of variability.

Certain other "mixing parameters" however, which could prove valuable in the design context considered above should be mentioned:

\[(\text{minimum dilution ratio}), \quad R_t = \frac{T_j - T_a}{T_{\text{max}} - T_a} \quad \ldots \quad \ldots \quad 5.34\]

\[(\text{minimum mixing efficiency}), \quad \eta_t = \frac{R_t}{(1 + V)} \quad \ldots \quad \ldots \quad 5.35\]

The (minimum) dilution ratio \(R_t\) is a measure of the degree of mixing at a particular section in a mixed flow field in terms of the highest temperature \(T_{\text{max}}\), usually at the water surface, recorded at that section. This parameter would be of particular use where a design specification (imposed for example, to satisfy some environmental constraint) was stated in such terms as:

"... outside of a specified mixing zone the temperature shall not exceed...."

The ranges of values taken by these two parameters are:

\[ (R_t)_{\text{max}} = 1 + V \]

(since \(q_j(T_j - T_a) = (q_a + q_j)(T_{\text{mix}} - T_a) \) in a completely
mixed flow i.e. where $T_{\text{max}} = T_{\text{mix}} = \frac{1}{T}$

$\left( R_t \right)_{\text{min}} = 1$

(at the section where the ports discharge).

mixing efficiency:

$\left( \eta_t \right)_{\text{max}} = 1$

(flow completely mixed)

$\left( \eta_t \right)_{\text{min}} = \frac{1}{(1 + V)}$

(at the discharge point).

In Figure 5.10 the mixing efficiency $\eta_t$ in the near field ($10 \leq X/H \leq 20$) is shown as a function of the square root of the mixing parameter, $\frac{F_j^2}{V^3}$, which was used in equations 5.11, 5.12, and 5.23 to differentiate between different mixing regimes. In addition to data from the Criterion 1 and Criterion 2 tests, data obtained by Goetz et al. (1972), and Chiang (1973), for large ambient to effluent discharge ratios, are included. Values of the ratio of the total to effluent discharge, $1 + \frac{q_a}{q_j}$, are shown as a third variable. The port spacing-to-depth ratio for all the data in Figure 5.10 is in the range $1 \leq L/H \leq 2.5$. There is evidently little dependence either on $1 + \frac{q_a}{q_j}$ except perhaps for highly stratified flows, or on $L/H$.

It can be shown from Figure 5.10 that as $q_a$ increases, corresponding to an increase in river discharge, the dilution at the point of least mixing $R_t$ increases also, even though there is a drop in efficiency $\eta_t$. Likewise, as $q_j$ decreases, corresponding to a decrease in effluent discharge, there is again an increase in dilution and
Figure 5.10  Efficiency of Mixing in the Near Field as a Function of Mixing Parameter.
drop in efficiency. However, for a given increase in $V$, due to either an increase in $q_a$ or a decrease in $q_j$, the gain in dilution tends to be significantly less when $q_j$ is decreased. For example, consider a case where the reference design calls for $F_j/V^{3/2} = 2$, for which the mixing efficiency according to Figure 5.10 is $\eta_t = 0.94$, and the dilution at the point of least mixing in a region 10 to 20 depths downstream from the diffuser pipe is $R_t/R_o$. For simplicity assume that $V = q_a/q_j >> 1$. Now let $V$ be doubled by doubling $q_a$. In this case $F_j/V^{3/2}$ is reduced by a factor of $2^{3/2} = 2.83$ to 0.71, for which the curve in Figure 5.10 indicates a reduction in $\eta_t$ to 0.71 and an increase in dilution of

$$\frac{R_t}{R_o} = (2)(0.71)/(0.94) = 1.51.$$ 

Now let $V$ be doubled by halving $q_j$, in which case $F_j/V^{3/2}$ is reduced by a factor of $2^{5/2} = 5.66$ to 0.35, resulting in a reduction of $\eta_t$ to 0.53 and an increase in dilution of

$$\frac{R_t}{R_o} = (2)(0.53)/(0.94) = 1.13.$$ 

As demonstrated in this example, Figure 5.10 can be useful for predicting the response in the performance of a diffuser pipe system to a change in operating conditions.

Another parameter of significance is heat recovery ratio:

$$R_h = \frac{\int_A u(T - T_a) \, dA}{(Q_a + Q_j)(T_{mix} - T_a)}$$  \hspace{2cm} 5.36

where the integration is carried out over the cross-sectional area of the channel. For a well-mixed flow equation 5.36 reduces to

$$R_h = \frac{T - T_a}{T_{mix} - T_a}$$  \hspace{2cm} 5.37
Except in small-scale models, heat loss in the near field region a short distance downstream from a diffuser pipe is generally negligible, so the heat recovery ratio is mainly relevant as a check on experimental measurements. A significant deviation from unity in a well-mixed flow would be indicative of errors in temperature or discharge measurements. Values of $R_h$ obtained in the present investigation are listed in Table A.11. With very few exceptions all values for well-mixed flows are within $\pm$ 5 percent of unity.
CHAPTER 6
CONCLUSIONS AND RECOMMENDATIONS

A. Summary of Main Findings

1. Mixed and Stratified Flows

A general classification criterion for use within (at least) the range covered by the present inquiry is presented:

General classification criterion:

\[
\frac{F_j^2 k_{m-3}^m}{\sqrt{V^m}} = C \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad 5.10
\]

where \( F_j^2 = \frac{\Delta \rho_j}{\rho g q_j} \)

\[k = \left( \frac{U_j}{U_a} \right)\]

\[V = \left( \frac{q_a}{q_j} \right)\]

\[C = \text{an appropriate constant or parameter.}\]

Experimental evidence is presented to suggest that \( m = 3 \); hence the general classification criterion becomes:

\[
F_j^2/V^3 = C \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad 6.1
\]

Four categories of flow are recognized; the appropriate criteria (based on \( m = 3 \)) follow:
Figure 6.1 Multi-port Diffusers in Open Channel Flow: Flow Classes and Regimes

Note: CVC values shown for "Intermediate-mixed" domain only.
Fully-mixed flow

\[ F_j^2 / V^3 > 15 \quad \ldots \quad \ldots \quad \ldots \quad 5.30 \]

Well-mixed flow

\[ F_j^2 / V^3 > 4 \quad \ldots \quad \ldots \quad \ldots \quad 5.12 \]

A "well-mixed" flow has a coefficient of (concentration) variation less than 0.03.

Intermediate-mixed flow

\[ F_j^2 / V^3 < 4 \quad \ldots \quad \ldots \quad \ldots \quad 5.12 \]

together with \[ F_j^2 / V^3 > 1 \quad \ldots \quad \ldots \quad \ldots \quad 5.11 \]

An "intermediate-mixed" flow is a mixed flow (i.e. not stratified) which has a coefficient of (concentration) variation greater than 0.03.

Stratified flow

\[ F_j^2 / V^3 < 1 \quad \ldots \quad \ldots \quad \ldots \quad 5.11 \]

A stratified flow (in the context of the present inquiry) is a flow system which exhibits the presence of ambient (concentration) conditions at bed level.

Lines separating these flow domains on the \( F_j - V/k \) plane are shown in Figure 6.1.

In the present inquiry, the "concentration" referred to above involved heated water; however, the criteria apply equally well to the (non-reactive) mixing of any liquid.
pollutant whose density differs from that of the receiving liquid.

In all cases, the criteria refer to the flow conditions well downstream (distance = 20 x flow depth approximately) from the discharge ports.

2. Mixing Distance for Non-stratified Flows

A general expression relating degree of mixing - as measured by the coefficient of (concentration) variation - to downstream distance (X) and the other variables was found (combining equations 5.20, 5.21).

\[ CV_x \left( \frac{L}{H-h} \right) = A \left( \frac{X}{L} \right)^N \ldots \ldots \ldots \ldots \quad 6.2 \]

where \( L = \) port spacing distance

\( H = \) ambient flow depth

\( h = \) port centerline height above bed

\( A,N \) are constants related to spacing and flow parameters.

The relationship of \( N \) to the spacing parameter (S) and the discharge ratio (V) is given in Figure 5.6; the relationship of \( A \) to \( S \) and the velocity ratio (k) is given in Figure 5.7.

Use can be made of these relationships together with the various classification criteria presented above to determine the "ultimate" degree of mixing (see definition of \( CV_c \) in Chapter 4, Section B (2.1)) which can be expected when a given multi-port
diffuser is subject to a given set of flow conditions. The main application of this information is in the investigation and design setting: APPENDIX C is devoted to such considerations.

(Experimentally determined values of $Cv_c$ around the "intermediate-mixed" flow domain have been entered on the general compilation, Figure 6.1, to aid in the design application of equation 6.2).

3. Flow Regimes

Cederwall's (1971) $F_j - V/k$ plane was divided into a number of flow regime sectors based on the collected observations of a number of investigators. These zones, with the classification criteria and $Cv_c$ data, combine to yield a formidable quantity of information about the mixing of multi-port diffuser jets in confined flowing environments.

4. Design of Multi-port Diffusers

4.1 A design Procedure

The main aim behind this study of confined flow mixing has been to produce criteria and guidelines for use in the design of multi-port diffusers. With the results of the investigation reported, it is now possible to concentrate attention on this area of the study.

In APPENDIX C is presented an outline design procedure and an example of its use to fix dimensions of the discharge system
(port diameter, port spacing) of a multi-port diffuser required to serve in the following situation:-

river discharge = 22,000 cfs
river depth = 20 ft
heated water discharge = 4,000 cfs
(designed) river temperature = 75°F
heated water temperature = 104°F

A trial diffuser 1800 ft long, having port centerline elevation θ = 20°, is investigated.

For "well-mixed" flow:-

port diameter = 3.0 ft;
port spacing = 16.1 ft (minimum).

For "fully-mixed" flow:-

port diameter = 3.0 ft;
port spacing = 25.1 ft (minimum).

The design procedure culminates in an estimate of the distance (from the diffuser) to the section where the (design) condition - "well-mixed", "fully-mixed" etc. - will be met; for the two cases investigated the results are:

"well-mixed" - 103 ft downstream from ports
"fully-mixed" - 140 ft downstream from ports.

While such accuracy as is implied by these results cannot be validly claimed, they do provide the designer, with, firstly, a yardstick by which he can assess the effectiveness
of alternate proposals and, secondly some quantitative information as to the order of distance required for the mixing process to take place.

4.2 Dilution Ratio and Mixing Efficiency

The dilution ratio at the point of least mixing, usually at the water surface, at a particular cross section,

\[ R_t = \frac{T_j - T_a}{T_{max} - T_a} \quad \ldots \quad \ldots \quad \ldots \quad 5.34 \]

and the mixing efficiency for that point,

\[ \eta_t = \frac{R_t}{(1 + V)} \quad \ldots \quad \ldots \quad \ldots \quad 5.35 \]

are useful concepts in designing diffuser pipe systems to satisfy those standards set forth by environmental protection agencies which require that the temperature (concentration) outside of a given mixing zone shall not exceed a specified limit. In Figure 5.10 \( \eta_t \), in the region a short distance downstream from the diffuser pipe (10 \( \leq \) X/H \( \geq \) 20), is related empirically to the mixing parameter \( F_j^2/V^3 \).

Using Figure 5.10 it is demonstrated that although a given increase in \( V = q_a/q_j \), due to either an increase in river discharge or a decrease in effluent discharge, causes a drop in efficiency, the dilution \( R_t \) increases in both cases, although by a lesser amount when \( q_j \) is decreased. This type of information is of value in predicting the response in the performance of a multi-port diffuser pipe system to a change in operating conditions.
B. Suggestions for Future Study

The region of the $F_j - V/k$ plane treated in the present study, although small, is significant in practical terms: it is the region which encompasses the discharge of cooling water into rivers. The program as reported here can, therefore, only hope to "scratch the surface" of such a vital problem; a series of in-depth studies is called for to answer the many questions raised. Some of the questions follow:-

1. Flow classification:

a) concerning the general flow classification equation:-

$$F_j^2 \frac{k^{m-3}}{V^m} = C \quad \ldots \quad \ldots \quad \ldots \quad 5.10$$

is $m$ constant for each criterion?

is $m$ the same for all criteria?

if $m$ is different for each criterion, what are its values?

b) The experimental results (see Chapter 5, Section A (1.2)) suggest that the boundary which divides the "stratified" and "intermediate-mixed" domains should be represented by
a family of lines in place of the single boundary offered in this report:

is this so and, if it is, what factors govern the positioning of these lines on the $F_j - (V/k)$ plane?

It is further suggested that a similar situation (although much less critical) obtains at the "intermediate-mixed"/"well-mixed" border:

is this so?

2. Mixing in a Confined Flow

A completely empirical relationship has been obtained:

$$CV_x \left( \frac{L}{H-h} \right) = A (X/L)^N \quad \ldots \quad \ldots \quad \ldots \quad 6.2$$

a) What theoretical justification can be found to substantiate the values of $A$ and $N$ obtained in the present inquiry?

What is the inter-relationship of $N$, $S$ and $V$ for values of $V < 4$ when flow is "well-mixed", "fully-mixed"?

Does the quantity $(-NS^4)$ tend to a constant value as is suggested by the present results?

What is the inter-relationship of $A$, $S$ and $(U_a/U_j)^2$ for values of $(U_a/U_j) > 0.07$?

b) There are probably many mixed flow applications where values of $CV_c > 0.03$ (i.e. "intermediate-mixed" flows) might be acceptable. With more information on values of $CV_c$ within


APPENDIX A

EXPERIMENTAL DATA, RESULTS AND PARAMETERS
APPENDIX A

EXPERIMENTAL DATA, RESULTS AND PARAMETERS

A. Sampling Study

1. Sampling Period Study (see Chapter 4, Section B (1.))

Temperature readings were taken at a particular section in a steady-state mixed flow; sampling periods 20 secs., 40 secs., 60 secs., 90 secs., 120 secs., were used. The readings were analyzed to determine the CV value for each time period for each of four grids. The results are shown in Table A.1

Assuming that the best estimate of CV at the section arises from Grid 1 (21 x 9) and sampling period 120 secs., the results show that sampling periods of either 60 secs., or 90 secs., may be used with either the 21x9 or 11x9 grids to yield CV values that differ from the "best estimate" by 1.07% or less.

Of these two sampling periods 60 secs., was chosen on the grounds of convenience.

<table>
<thead>
<tr>
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<td>CV Values for Sampling Period Study</td>
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<th>60 secs.</th>
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<td>CV % diff</td>
<td>CV % diff</td>
<td>CV % diff</td>
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<td>1.28</td>
<td>3.84</td>
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* % difference between test CV value and 'best estimate' value.
2. Grid Selection Study (see Chapter 4, Section B (1.))

Sixteen sets of temperature data were taken (21x9 grid) at various sections in seven different flow systems; sampling period = 60 secs., was used. CV values were determined using four grids to analyze the (same) basic data. The results are presented in Table A.2

Assuming that the best estimate of CV in each case is the value given by the 21x9 grid, the results demonstrate that:-

(i) the 11x9 grid (grid 2) is far superior to grids 3 and 4,
(ii) the results given by grid 2 are quite close (within 1%, on average) to the "best estimate" values.

For these reasons, grid 2 (11x9) was used for most of the experimental program.

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<th>GRID 2 11x9</th>
<th>GRID 3 21x5</th>
<th>GRID 4 11x5</th>
<th>% Diff. between grid value and Grid 1 value: (2-1) (3-1) (4-1)</th>
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B. Mixing Investigation Data, Results and Parameters
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3-pipe tests

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</tr>
<tr>
<td>19</td>
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<tr>
<td>20</td>
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</tbody>
</table>

2-pipe tests

| 21 | 0.0390 | .405 | 0.096  | 73.67 | 0.0067 | 0.0060 | 1.12  | 35.56 | 1.000 | 0.645 |
| 22 | 0.0389 | .405 | 0.096  | 73.61 | 0.0067 | 0.0060 | 1.12  | 11.50 | 1.000 | 0.173 |
| 23 | 0.0395 | .405 | 0.098  | 73.11 | 0.0067 | 0.0060 | 1.12  | 20.94 | 1.000 | 0.337 |
| 24 | 0.055  | .405 | 0.136  | 72.11 | 0.0065 | 0.0060 | 1.08  | 32.44 | 1.000 | 0.563 |
| 25 | 0.0603 | .405 | 0.149  | 73.54 | 0.0071 | 0.0060 | 1.18  | 32.57 | 1.000 | 0.580 |
| 26 | 0.0488 | .405 | 0.111  | 73.94 | 0.0071 | 0.0060 | 1.18  | 15.73 | 1.000 | 0.248 |
| 27 | 0.0453 | .405 | 0.112  | 74.66 | 0.0066 | 0.0060 | 1.10  | 32.37 | 1.000 | 0.582 |
| 28 | 0.0453 | .405 | 0.112  | 74.80 | 0.0067 | 0.0060 | 1.12  | 14.69 | 1.000 | 0.230 |
| 29 | 0.0488 | .405 | 0.111  | 74.59 | 0.0096 | 0.0060 | 1.60  | 32.43 | 1.000 | 0.582 |
| 30 | 0.0447 | .405 | 0.111  | 74.63 | 0.0060 | 0.0060 | 1.00  | 32.63 | 1.000 | 0.587 |
| 31 | 0.0447 | .405 | 0.111  | 74.57 | 0.0060 | 0.0060 | 1.00  | 18.74 | 1.000 | 0.303 |
Table A.4
Mixing Investigation Parameters - Tests 1 – 32

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<th>$\frac{1}{k} = \frac{q_j}{q_a}$</th>
<th>$\frac{S}{U_a/U_j}$</th>
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4-pipe tests

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<th>$\frac{1}{k} = \frac{q_j}{q_a}$</th>
<th>$\frac{S}{U_a/U_j}$</th>
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2-pipe tests

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Basic Criterion 1 Test Results - Tests 1-32

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4-pipe tests

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Table A.6

(X/L) and CV (L/H-h) values - Tests 1-32
Table A.7

Basic Criterion 2 Test Data - Tests 33-40

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### Table A.9

**Flow Regimes -- Other Sources**

**Cederwall (1971)**

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**Harleman et al (1968):**

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**Jain et al (1971):**

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**Goetz et al (1972):**

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*It is presumed that no upstream thermal wedge was present, hence the (bracketed) class C.
Table A.10

Dilution and Mixing Efficiency Data

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APPENDIX B

GRAPHS FOR CRITERION 1 AND CRITERION 2 TESTS
Figure B.1 CV(L/(H-h)) vs (X/L) - 2-pipe Tests
Figure B.2 CV(L/(H-h)) vs (X/L) - 3-pipe Tests
Figure B.3 CV(L/(H-h)) vs (X/L) - 4-pipe Tests
Figure B.4 Criterion 2 Tests - Tests 33, 34, 35
Figure B.5 Criterion 2 Tests - Tests 36, 37
Figure B.6 Criterion 2 Tests - Tests 38, 39, 40
APPENDIX C

DESIGN APPLICATION
APPENDIX C

DESIGN APPLICATION

A. General Procedure to test Trial Diffuser

Given: trial diffuser length (l) ... ... ... ... D1
port centerline elevation, \( \Theta = 20^\circ \) ... ... ... ... D2

Given: discharge \((q_a)\) and depth of receiving water flow
across diffuser \((H)\),

\[ q_a = \frac{Q_a}{l} \quad ... ... \quad D3 \]
and \[ U_a = q_a/H \quad ... ... \quad D4 \]

Given: diffuser discharge \((q_j)\),

\[ q_j = \frac{Q_j}{l} \quad ... ... \quad D5 \]

Given: design heated water temperature excess (above ambient), \( \Delta t = T_j - T_a \),

\[ \Delta \rho/\rho \quad \text{(from tables)} \quad ... \quad D6 \]

From D3, D5,

\[ V = \left(\frac{q_a}{q_j}\right) \quad ... ... \quad D7 \]

Steps in procedure:

1. Select mixed flow condition required - "fully-mixed",
    "well-mixed", etc. - and apply appropriate criterion (Using D7 above) to determine required \( F_j^2 \).
2. Determine jet velocity \( U_j \) from \( F_j^2 \);

\[
F_j = \frac{U_j^3}{a^4} \quad \text{and} \quad V/k = \frac{U_a q_a}{U_j q_j}
\]

and check flow regime on Figure 6.1; if unsatisfactory e.g., if upstream thermal wedge predicted which cannot be tolerated, alter trial length \( L \) and repeat steps 1 and 2; if satisfactory, proceed.

3. Knowing \( q_j \) and \( U_j \) determine \( (\text{port area})/L \):

\[
q_j/U_j = A_0/L
\]

At this point either a trial port area \( A_0 \) or a trial port spacing distance \( L \) must be introduced since the value of "\( L \)" is required in subsequent calculations.

hence \( S = A_0/L^2 \)

4. Knowing \( 1/V = q_j/q_a \), find \(-NS^4\) from Figure 5.6; hence \( N \).

5. Knowing \( 1/k = U_a/U_j \) find \( AS \) from Figure 5.7; hence \( A \).

6. Using an appropriate value for \( CV_c \) consistent with the selection made in step 1 above, e.g.

use \( CV_c = 0.01 \) for "fully-mixed" flow,

use \( CV_c = 0.03 \) for "well-mixed" flow,

substitute for \( A,N \) into equation 6.2:

\[
CV_c \left( \frac{L}{H-h} \right) = A \left( \frac{X_c/L}{N} \right)
\]
to determine $X_c$, the downstream distance to the flow cross section where $CV_x = CV_c$; for preliminary design trials assume $(H-h) = 0.9H$.

**B. Example of Diffuser Test Procedure**

Given: diffuser length $l = 1800$ ft

port centerline: $\Theta = 20^\circ$

Given: $Q_a = 18,000$ cfs, $H = 20$ ft., $q_a = 10$ cfs/ft

$U_a = 0.5$ f.p.s.

Given: $Q_j = 4,000$ c.f.s; $q_j = 2.22$ cfs/ft

**Given:** $T_a = 75^\circ$F, $T_j = 104^\circ$F; $\Delta \rho/\rho = 0.0050$

1. Select "well-mixed" criterion, equation

$$F_j^2/V^3 > 4 \ldots \ldots \ldots 5.12$$

$$V = q_a/q_j = 10/2.22 = 4.5$$

then (minimum) $F_j^2 = 4 \times 4.5^3 = 366$

2. $F_j = \frac{U_j^3}{\rho \Delta \rho g q_j} = 366$, hence $U_j = 5.07$ fps

$F_a = \frac{U_a^3}{\rho \Delta \rho g q_j} = 0.35$

$$V/k = \frac{U_a q_a}{U_j q_j} = 0.44$$
Flow regime: Zone A2 (well-mixed)

i.e. substantial upstream thermal wedge with downstream flow "well-mixed"
(satisfactory).

3. \( \frac{\text{Port area}}{L} = \frac{q_j}{U_j} = 0.438 \text{ ft}^2/\text{ft} \)

let trial port area = 7.07 ft\(^2\) (3ft diameter)
then \( L = 16.1 \text{ ft} \)
\( S = 0.0272 \)

4. \( \frac{1}{V} = \frac{q_j}{q_a} = 0.222 \) hence \(-NS^1\)

hence \( N = -2.71 \)

5. \( \frac{1}{k} = \frac{U_a}{U_j} = 0.0986 \) hence \( AS = 0.115 \)

hence \( A = 4.22 \)

6. Use \( CV_c = 0.03 \) (for "well-mixed" flow)

then \( 0.03 \left( \frac{16.1}{0.9 \times 20} \right) = 4.22 \) \( (X_c/16.1) \)

hence distance to section where flow is "well-mixed"
(i.e. where \( CV_x = 0.03 \)) is

\( X_c = 103 \text{ ft} \)

this flow system is "just" well-mixed (since \( F_j^2 \) (minimum)
was adopted in step 1); the port spacing distance obtained here \( (L = 16.1 \text{ ft}) \) thus represents a minimum value for 3 ft diameter ports.

Similarly, for a design requirement of "fully-mixed" flow
with 3.0 ft diameter ports, a diffuser having the following
properties results:

\[ l = 1800 \text{ ft} \]
\[ L_{\text{min}} = 25.1 \text{ ft} \]
\[ \text{port diam} = 3.0 \text{ ft} \]
\[ F_j = 0.35 \text{ Zone A2} \]
\[ V/k = 0.28 \text{ upstream thermal wedge} \]
for \[ CV_c = 0.01, X_c = 140 \text{ ft} \]

It is possible, using the procedure outlined above to design a "slot jet" type of diffuser i.e. a diffuser which discharges through a row of closely spaced ports.

For such a case, with design conditions as before, the result is:

diffuser length, \[ l = 1800 \text{ ft} \]

\[ L_{\text{min}} = 1.0 \text{ ft} \]
\[ \text{port diam} = 0.6 \text{ ft} \]
\[ F_j = 0.35 \text{ Zone A2} \]
\[ V/k = 0.28 \text{ upstream thermal wedge} \]
for \[ CV_c = 0.01, X_c = 53 \text{ ft} \]

Application of the confined flow relationships developed in the present inquiry to a case where jet interaction undoubtedly dominates, must however be ruled invalid and can, at best, be considered as giving an indication of a possible design only.