**Handout: Turbulent Mixing Layer**

- Flow between two uniform parallel streams of different velocities $U_h > U_l \geq 0$

\[ U(y<0,x=0)=U_l, \quad U(y>0,x=0)=U_h \]

![Fig. 5.14 from Pope’s Turbulent Flows book](image)

- Two velocities $\rightarrow$ new parameter $U_l / U_h$

\[ \frac{\langle U(x,t) \rangle}{U_h} = f(Re, \frac{y}{x}, \frac{U_l}{U_h}) \]
where <> denotes a mean (Reynolds averaged) quantity

→ Mixing layer is self-similar

- **Characteristic velocity**
  \[ U_c = \frac{1}{2}(U_h + U_l) \]

- **Velocity difference**
  \[ U_s = U_h - U_l \]

- **Characteristic mixing layer width**
  With \( y_\alpha(x) \) defined by \( \langle U(x, y_\alpha(x)) \rangle = U_l + \alpha U_s \)
  \[ \delta(x) = y_{0.9}(x) - y_{0.1}(x) \]

- **Reference lateral position**
  \[ \bar{y}(x) = \frac{1}{2}(y_{0.9}(x) + y_{0.1}(x)) \]

- **Scaled cross-stream coordinate**
  \[ \xi = \frac{y - \bar{y}(x)}{\delta(x)} \]

- **Scaled velocity**
  \[ f(\xi) = \frac{\langle U > -U_c}{U_s} \]
  \[ f(\pm\infty) = \pm \frac{1}{2} \]
  \[ f(\pm \frac{1}{2}) = \pm 0.4 \]
Fig. 5.21. A sketch of the mean velocity (U) against y, and of the scaled mean velocity profile f(ξ), showing the definitions of y₀₁, y₀.₉, and δ.

Fig. 5.22. Scaled velocity profiles in a plane mixing layer. Symbols, experimental data of Champagne et al. (1976) (○, x = 39.5 cm; □, x = 49.5 cm; ◊, x = 59.5 cm); line, error-function profile (Eq. (5.224)) shown for reference.

- Flow not symmetric around y=0
Spreading Rate

\[ \frac{d\delta(x)}{dx} = \text{const} = \frac{U_s}{U_c} \]

\[ \Rightarrow S = \frac{U_c}{U_s} \frac{d\delta(x)}{dx} \quad S \approx 0.06 - 0.11 \text{ is independent of } \frac{U_s}{U_c} \]

The variation in the range of recorded values depends on the state of the flow as it leaves the splitter plate.

Turbulent Kinetic Energy Flow Rate

\[ K(x) \sim \int_{-\infty}^{\infty} <U> k dy \sim U_c U_s^2 \delta \sim x \quad \text{as} \quad \delta \sim x \]

\[ K(x) \text{ increasing with } x \]

\[ \Rightarrow P > \varepsilon \quad (\text{Rogers & Moser (1994)}: \frac{P}{\varepsilon} = 1.4) \]
Statistics for self-similar plane mixing layer

Fig. 5.25. Scaled Reynolds-stress profiles in self-similar plane mixing layers. Symbols, experiment of Bell and Mehta (1990) \((U_i/U_h = 0.6)\); solid line, DNS data for the temporal mixing layer (Rogers and Moser 1994).

Temporal Mixing Layer

- Limit:
  \[
  \frac{U_s}{U_c} \rightarrow 0 \quad \text{or} \quad \frac{U_i}{U_h} \rightarrow 0
  \]

- Boundary layer equation for self-similar mixing layer
(g = \langle uv \rangle / U_s^2, f' = df/d\xi, g' = dg/d\xi)

\[
\left( \frac{U_c}{U_s} \frac{d\delta}{dx} \right) \left( \xi + \frac{U_s}{U_c} \int_0^\xi f(\xi')d\xi' \right)f' = g'
\]

reduces to

\[
U_c \frac{\partial \langle U \rangle}{\partial x} = -\frac{\partial \langle uv \rangle}{\partial y}
\]

with \( \tau = x / U_c \)

An observer traveling in the x direction at speed \( U_c \) sees two streams moving to right and left with velocities 0.5\( U_s \) and -0.5\( U_s \) at \( \pm \infty \). Gradients of mean quantities in the x direction are vanishingly small (of order of \( U_s / U_c \)) compared with gradients in y direction. The thickness of the mixing layer grows it time at rate \( SU_s \). Thus, in the moving frame, as \( U_s / U_c \) tends to zero, the flow becomes statistically one-dimensional and time-dependent. It is called the temporal mixing layer and it is statistically symmetric about \( y = 0 \).